MODERN ALGEBRA

CLASS ASSIGNMENT

DECEMBER - 2014

PART - B

1. The number of conjugacy classes in the permutation group S_6 is
1. 12 2. 11 3. 10 4. 6

2. Find the degree of the field extension $\mathbb{Q}\left(\sqrt{2},\sqrt[4]{2},\sqrt[8]{2}\right)$ over $\mathbb{Q}$.
1. 4 2. 8 3. 14 4. 32

3. Let G be the Galois group of a field with nine elements over its subfield with three elements. Then the number of orbits for the action of G on the field with nine elements is
1. 3 2. 5 3. 6 4. 9

4. The number of surjective maps from a set of 4 elements to a set of 3 elements is
1. 36 2. 64 3. 69 4. 81

5. In the group of all invertible 4×4 matrices with entries in the field of 3 elements, any 3-Sylow subgroup has cardinality
1. 3 2. 81 3. 243 4. 729

PART - C

6. Let G be a nonabelian group. Then, its order can be
1. 25 2. 55 3. 125 4. 35

7. Let $\mathbb{R}[x]$ be the polynomial ring over $\mathbb{R}$ in one variable. Let $I \subseteq \mathbb{R}[x]$ be an ideal. Then
1. I is a maximal ideal if and only if I is a non-zero prime ideal
2. I is a maximal ideal if and only if the quotient ring $\mathbb{R}[x]/I$ is isomorphic to $\mathbb{R}$.
3. I is a maximal ideal if and only if $I=(f(x))$, where f(x) is a non–constant irreducible polynomial over $\mathbb{R}$
4. I is a maximal ideal if and only if there exists a non-constant polynomial $f(x)\in I$ of degree ≤ 2

8. Let G be a group of order 45. Then
1. G has an element of order 9
2. G has a subgroup of order 9
3. G has a normal subgroup of order 9
4. G has a normal subgroup of order 5

9. Which of the following is/are true?
1. Given any positive integer n, there exists a field extension of $\mathbb{Q}$ of degree n.
2. Given a positive integer n, there exist fields F and K such that $F \subseteq K$ and K is Galois over F with [K:F]=n.
3. Let K be a Galois extension of $\mathbb{Q}$ with $[K:\mathbb{Q}] = 4$. Then there is a field L such that $K \supseteq L \supseteq \mathbb{Q}$, $[L:\mathbb{Q}] = 2$ and L is a Galois extension of $\mathbb{Q}$.
4. There is an algebraic extension K of $\mathbb{Q}$ such that $[K:\mathbb{Q}]$ is not finite.

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PART - B

10. Up to isomorphism, the number of abelian groups of order 108 is:
1. 12 2. 9
3. 6 4. 5

11. Let D be the set of tuples $(w_1,\ldots\ldots,w_{10})$,where $w_i \in \{1,2,3\}, 1\le i \le 10$ and $w_i + w_{i+1}$ is an even number for each i with $1\le i \le 9$. Then the number of elements in D is.
1. $2^{11}+1$ 2. $2^{10}+1$
3. $3^{10}+1$ 4. $3^{11}+1$

12. The number of subfields of a field of cardinality 2^{100} is
1. 2 2. 4
3. 9 4. 100

13. Let R be the ring $\mathbb{Z}[x]/((x^2+x+1)(x^3+x+1))$ and I be the ideal generated by 2 in R. What is the cardinality of the ring R?
1. 27 2. 32
3. 64 4. Infinite.

PART - C

14. Which of the following polynomials are irreducible in the ring $\mathbb{Z}[x]$ of polynomials in one variable with integer coefficients?
1. $x^2 – 5$
2. $1+(x+1) + (x+1)^2+ (x+1)^3+ (x+1)^4$
3. $1+ x + x^2+ x^3 + x^4$
4. $1+ x + x^2+ x^3$

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15. Determine which of the following polynomials are irreducible over the indicated rings.

1. $x^5 - 3x^4 + 2x^3 - 5x + 8$ over $\mathbb{R}$.
2. $x^3 + 2x^2 + x + 1$ over $\mathbb{Q}$.
3. $x^3 + 3x^2 - 6x + 3$ over $\mathbb{Z}$.
4. $x^4 + x^2 + 1$ over $\mathbb{Z}/2\mathbb{Z}$.

16. Let $\sigma:\{1,2,3,4,5\} \to \{1,2,3,4,5\}$ be a permutation (one - to – one and onto function) such that $\sigma^{-1}(j) \le \sigma(j)\ \forall j, 1 \le j \le 5$.
Then which of the following are true?

1. $\sigma \circ \sigma(j) = j$ for all j, $1 \le j \le 5$.
2. $\sigma^{-1}(j) = \sigma(j)$ for all j, $1 \le j \le 5$.
3. The set $\{k: \sigma(k) \ne k\}$ has an even number of elements.
4. The set $\{k: \sigma(k) = k\}$ has an odd number of elements.

17. If x,y and z are elements of a group such that xyz = 1, then

1. yzx = 1
2. yxz = 1
3. zxy = 1
4. zyx = 1

18. Which of the following primes satisfy the congruence $a^{24} \equiv 6a + 2 \bmod 13$?

1. 41 2. 47 3. 67 4. 83

19. Let C([0,1]) be the ring of all real valued continous functions on [0,1]. Which of the following statements are true?

1. C([0,1]) is an integral domain.
2. The set of all functions vanishing at 0 is a maximal ideal.
3. The set of all functions vanishing at both 0 and 1 is a prime ideal.
4. If $f \in C([0,1])$ is such that $(f(x))^n = 0$ for all $x \in [0,1]$ for some n > 1, then f(x) = 0 for all $x \in [0,1]$.

20. Which of the following cannot be the class equation of a group of order 10?

1. 1 + 1 + 1 + 2 + 5 = 10.
2. 1 + 2 + 3 + 4 = 10.
3. 1 + 2 + 2 + 5 = 10.
4. 1 + 1 + 2 + 2 + 2 + 2 = 10.

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PART - B

21. Which of the following is an irreducible factor of $x^{12} - 1$ over $\mathbb{Q}$?

1. $x^8 + x^4 + 1$.
2. $x^4 + 1$
3. $x^4 - x^2 + 1$.
4. $x^5 - x^4 + x^3 - x^2 + x - 1$.

22. Let R be a Euclidean domain such that R is not a field. Then the polynomial ring R[X] is always

1. a Euclidean domain
2. a principal ideal domain, but not a Euclidean domain.
3. a unique factorization domain, but not a principal ideal domain.
4. not a unique factorization domain.

23. What is the total number of positive integer solutions to the equation

$(x_1 + x_2 + x_3)(y_1 + y_2 + y_3 + y_4) = 15$?

1. 1 2. 2 3. 3 4. 4

24. A group G is generated by the elements x, y with the relations $x^3 = y^2 = (xy)^2 = 1$. The order of G is

1. 4. 2. 6. 3. 8. 4. 12.

25. Let G be a simple group of order 60. Then

1. G has six Sylow-5 subgroups
2. G has four Sylow-3 subgroups.
3. G has a cyclic subgroup of order 6.
4. G has a unique element of order 2.

PART - C

26. Let $\omega = \cos\frac{2\pi}{10} + i\sin\frac{2\pi}{10}$.

Let $K = \mathbb{Q}(\omega^2)$ and let $L = \mathbb{Q}(\omega)$. Then

1. $[L : \mathbb{Q}] = 10$
2. $[L : K] = 2$
3. $[K : \mathbb{Q}] = 4$
4. $L = K$

27. Let a_n denote the number of those permutations σ on {1, 2, …, n} such that σ is a product of exactly two disjoint cycles. Then:

1. $a_5 = 50$
2. $a_4 = 14$
3. $a_5 = 40$
4. $a_4 = 11$

28. Which of the following quotient rings are fields?

1. $F_3[X]/(X^2+X+1)$, where F_3 is the finite field with 3 elements.
2. $\mathbb{Z}[X]/(X - 3)$
3. $\mathbb{Q}[X]/(X^2 + X + 1)$
4. $F_2[X]/(X^2 + X + 1)$ where F_2 is the finite field with 2 elements.

29. Which of the following intervals contains an integer satisfying the following three congruences:
$x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$ and $x \equiv 4 \pmod{11}$.

1. [401, 600]
2. [601, 800]
3. [801, 1000]
4. [1001, 1200]

30. Let A denote the quotient ring $\mathbb{Q}[X]/(X^3)$. Then
1. There are exactly three distinct proper ideals in A.
2. There is only one prime ideal in A.
3. A is an integral domain
4. Let f, g be in $\mathbb{Q}[X]$ such that $\overline{f}.\overline{g}=0$ in A. Here $\overline{f}$ and $\overline{g}$ denote the image of f and g respectively in A. Then f(0).g(0)= 0.

31. For n ≥ 1, let $(\mathbb{Z}/n\mathbb{Z})^*$ be the group of units of $(\mathbb{Z}/n\mathbb{Z})$. Which of the following groups are cyclic?

1. $(\mathbb{Z}/10\mathbb{Z})^*$ 2. $(\mathbb{Z}/2^3\mathbb{Z})^*$
3. $(\mathbb{Z}/100\mathbb{Z})^*$ 4. $(\mathbb{Z}/163\mathbb{Z})^*$

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PART - B

32. Which of the following statements is FALSE?
There exists an integer x such that:
1. x ≡ 23 mod 1000 and z ≡ 45 mod 6789
2. x ≡ 23 mod 1000 and z ≡ 54 mod 6789
3. x ≡ 32 mod 1000 and z ≡ 54 mod 9876
4. x ≡ 32 mod 1000 and z ≡ 44 mod 9876

33. Let $G=(Z/25Z)^*$ be the group of units (i.e. the elements that have a multiplicative inverse) in the ring (Z/25Z). Which of the following is a generator of G?

1. 3 2. 4
3. 5 4. 6

34. Let p≥5 be a prime. Then
1. $F_p \times F_p$ has at least five subgroups of order p.
2. Every subgroup of $F_p \times F_p$ is of the form $H_1 \times H_2$ where H_1, H_2 are subgroup of F_p.
3. Every subgroup of $F_p \times F_p$ is an ideal of the ring $F_p \times F_p$
4. The ring $F_p \times F_p$ is a field.

35. Let p be a prime number. How many distinct sub – rings (with unity) of cardinality p does the field F_{p^2} have?

1. 0 2. 1 3. p 4. p^2

PART - C

36. Consider the symmetric group S_{20} and its subgroups A_{20} consisting of all even permutations. Let H be a 7-Sylow subgroup of A_{20}. Pick each correct statement from below.
1. |H| = 49.
2. H must be cyclic.
3. H is a normal subgroup of A_{20}.
4. Any 7-Sylow subgroup of S_{20} is a subset of A_{20}.

37. Let R be a commutative ring with unity, such that R[X] is a UFD. Denote the ideal (X) of R[X] by I. Pick each correct statement from below.
1. I is prime.
2. If I is maximal, then R[X] is a PID.
3. If R[X] is a Euclidean domain, then I is maximal.
4. If R[X] is a PID, then it is a Euclidean domain.

38. Let G be a finite abelian group of order n. Pick each correct statement from below.
1. If d divides n, there exists a subgroup of G of order d.
2. If d divides n, there exists an element of order d in G.
3. If every proper subgroup of G is cyclic, then G is cyclic.
4. If H is a subgroup of G, there exists a subgroup N of G such that G/N ≅ H.

39. Let p be a prime, Pick each correct statement from below. Up to isomorphism.
1. There are exactly two abelian groups of order p^2.
2. There are exactly two groups of order p^2.
3. There are exactly two commutative rings of order p^2.
4. There is exactly one integral domain of order p^2.

40. Let $f(x) \in \mathbb{Z}[x]$ be a polynomial of degree ≥ 2. Pick each correct statement from below.
1. If f(x) is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{Q}[x]$.
2. If f(x) is irreducible in $\mathbb{Q}[x]$, then it is irreducible in $\mathbb{Z}[x]$.
3. If f(x) is irreducible in $\mathbb{Z}[x]$, then for all primes p the reduction $\overline{f(x)}$ of f(x) modulo p is irreducible in $F_p[x]$.

4. If f(x) is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{R}[x]$.

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PART - B

41. (n-1)!≡ -1 (mod n). We can conclude that
1. $n = p^k$ where p is prime, k > 1.
2. n = pq where p and q are distinct primes.
3. n = pqr where p, q, r are distinct primes.
4. n = p where p is a prime.

42. Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is true?
1. There exists a finite group which is not a subgroup of S_n for any $n \geq 1$.
2. Every finite group is a subgroup of A_n for some $n \geq 1$.
3. Every finite group is a quotient of A_n for some $n \geq 1$.
4. No finite abelian group is a quotient of S_n for n > 3.

PART - C

43. Consider the following subsets of the group of 2×2 non-singular matrices over $\mathbb{R}$:
G=$\left\{\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R}, ad = 1\right\}$
H=$\left\{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R}\right\}$.
Which of the following statements are correct?
1. G forms a group under matrix multiplication.
2. H is a normal subgroup of G.
3. The quotient group G/H is well-defined and is Abelian.
4. The quotient group G/H is well defined and is isomorphic to the group of 2×2 diagonal matrices (over $\mathbb{R}$) with determinant 1.

44. Let $\mathbb{C}$ be the field of complex numbers and $\mathbb{C}^*$ be the group of non zero complex numbers under multiplication. Then which of the following are true?
1. $\mathbb{C}^*$ is cyclic.
2. Every finite subgroup of $\mathbb{C}^*$ is cyclic.
3. $\mathbb{C}^*$ has finitely many finite subgroups.
4. Every proper subgroup $\mathbb{C}^*$ is cyclic.

45. Let R be a finite non-zero commutative ring with unity. Then which of the following statements are necessarily true?
1. Any non-zero element of R is either a unit or a zero divisor.
2. There may exist a non-zero element of R which is neither a unit nor a zero divisor.
3. Every prime ideal of R is maximal.
4. If R has no zero divisors then order of any additive subgroup of R is a prime power.

46. Which of the following statements are true?
1. $\mathbb{Z}$ is a principle ideal domain.
2. $\mathbb{Z}[x,y]$ / <y+1> is a unique factorization domain.
3. If R is a principle ideal domain and p is a non-zero prime ideal, then R/p has finitely many prime ideals.
4. If R is a principle ideal domain, then any subring of R containing 1 is again a principal ideal domain.

47. Let R be a commutative ring with unity and R[x] be the polynomial ring in one variable. For a non zero $f = \sum_{n=0}^{N} a_n x^n$, define $\omega(f)$ to be the smallest n such that $a_n \neq 0$. Also $\omega(0) = +\infty$. Then which of the following statements is/are true?
1. $\omega(f+g) \geq \min(\omega(f), \omega(g))$.
2. $\omega(fg) \geq \omega(f) + \omega(g)$.
3. $\omega(f+g) = \min(\omega(f), \omega(g))$, if $(\omega(f) \neq \omega(g)$.
4. $\omega(fg) = \omega(f) + \omega(g)$, if R is an integral domain.

48. Let $\mathbf{F}_2$ be the finite field of order 2. Then which of the following statements are true?
1. $\mathbf{F}_2[x]$ has only finitely many irreducible elements.
2. $\mathbf{F}_2$ [x] has exactly one irreducible polynomial of degree 2.
3. $\mathbf{F}_2[x]$ / <x^2 + 1> is a finite dimensional vector space over $\mathbf{F}_2$.
4. Any irreducible polynomial in $\mathbf{F}_2[x]$ of degree 5 has distinct roots in any algebraic closure of $\mathbf{F}_2$.

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PART - B

49. Consider the ideal $I=(x^2+1,y)$ in the polynomial ring $\mathbb{C}[x,y]$. Which of the following statements is true?
1. I is a maximal ideal
2. I is a prime ideal but not a maximal ideal
3. I is a maximal ideal but not a prime ideal
4. I is neither a prime ideal nor a maximal ideal

PART - C

50. For an integer n ≥2, let S_n be the permutation group on n letters and A_n the alternating group. Let $\mathbb{C}^*$ be the group of non–zero complex numbers under multiplication. Which of the following are correct statements?
1. For every integer n ≥ 2, there is a non trivial homomorphism $\chi : S_n \to \mathbb{C}^*$.
2. For every integer n ≥ 2, there is a unique nontrivial homomorphism $\chi : S_n \to \mathbb{C}^*$
3. For every integer n ≥ 3, there is a nontrivial homomorphism $\chi : A_n \to \mathbb{C}^*$
4. For every integer n ≥ 5, there is a nontrivial homomorphism $\chi : A_n \to \mathbb{C}^*$

51. Let R={f:{1,2,...,10}→$\mathbb{Z}_2$} be the set of all $\mathbb{Z}_2$ –valued functions on the set {1,2,...,10} of the first ten positive integers. Then R is commutative ring with pointwise addition and pointwise multiplication of functions. Which of the following statements are correct?
1. R has a unique maximal ideal
2. every prime ideal of R is also maximal
3. Number of proper ideals of R is 511
4. every element of R is idempotent

52. Which of the following rings are principal ideal domains (PID)?

1. **Q** [x] 2. $\mathbb{Z}$[x]
3. ($\mathbb{Z}/6\mathbb{Z}$)[x] 4. ($\mathbb{Z}/7\mathbb{Z}$)[x]

53. Let G be a group of order 125. Which of the following statements are necessarily true?
1. G has a non-trivial abelian subgroup
2. The centre of G is a proper subgroup
3. The centre of G has order 5
4. There is a subgroup of order 25

54. Let R be a non-zero ring with identity such that $a^2=a$ for all a∈R. Which of the following statements are true?
1. There is no such ring
2. 2a=0 for all a∈R
3. 3a=0 for all a∈R
4. $\mathbb{Z}/2\mathbb{Z}$ is a subring of R

55. Which of the following polynomials are irreducible in $\mathbb{Z}$[x]?

1. $x^4+10x+5$ 2. x^3-2x+1
3. x^4+x^2+1 4. x^3+x+1

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PART - B

56. Let f : $\mathbb{Z} \to (\mathbb{Z}/4\mathbb{Z}) \times (\mathbb{Z}/6\mathbb{Z})$ be the function f(n) = (n mod 4, n mod 6). Then
1. (0 mod 4, 3 mod 6) is in the image of f
2. (a mod 4, b mod 6) is in the image of f, for all even integers a and b
3. image of f has exactly 6 elements
4. kernel of f = 24$\mathbb{Z}$

57. The group S_3 of permutations of {1, 2, 3} acts on the three dimensional vector space over the finite field $\mathbf{F}_3$ of three elements, by permuting the vectors in basis {e_1,e_2,e_3} by σ. $e_i = e_{\sigma(i)}$, for all $\sigma \in S_3$. The cardinality of the set of vectors fixed under the above action is

1. 0 2. 3 3. 9 4. 27

58. Let R be a subring of $\mathbb{Q}$ containing 1. Then which of the following is necessarily true?
1. R is a principal ideal domain (PID)
2. R contains infinitely many prime ideals
3. R contains a prime ideal which is not a maximal ideal
4. for every maximal ideal m in R, the residue field R/m is finite

PART - C

59. Let G be a finite abelian group and a,b∈G with order(a) = m, order(b) =n. Which of the following are necessarily true ?
1. order (ab) = mn
2. order (ab) = lcm(m,n)
3. there is an element of G whose order is lcm (m,n)
4. order (ab)=gcd(m,n)

60. Which of the following rings are principal ideal domains (PIDs) ?
1. $\mathbb{Z}[X]/\langle X^2 + 1\rangle$
2. $\mathbb{Z}[X]$
3. $\mathbb{C}[X,Y]$
4. $\mathbb{R}[X,Y]/\langle X^2 + 1, Y\rangle$

61. For any prime number p, let A_p be the set of integers $d\in\{1,2,...,999\}$ such that the power of p in the prime factorisation of d is odd. Then the cardinality of
1. A_3 is 250 2. A_5 is 160
3. A_7 is 124 4. A_{11} is 82

62. Let $z = e^{\frac{2\pi i}{7}}$ and let $\theta = z + z^2 + z^4$. Then
1. $\theta \in \mathbb{Q}$
2. $\theta \in \mathbb{Q}\left(\sqrt{D}\right)$ for some D>0
3. $\theta \in \mathbb{Q}\left(\sqrt{D}\right)$ for some D<0
4. $\theta \in i\,\mathbb{R}$

63. Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to
1. the cyclic group of order 6
2. the permutation group on {1,2,3}
3. the permutation group on {1,2,3,4,5,6}
4. the permutation group on {1}

JUNE - 2018

PART – B

64. Let S_7 denote the group of permutations of the set {1,2,3,4,5,6,7}. Which of the following is true?
1. There are no elements of order 6 in S_7
2. There are no elements of order 7 in S_7
3. There are no elements of order 8 in S_7
4. There are no elements of order 10 in S_7

65. The number of group homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is
1. zero 2. one
3. five 4. Ten

PART – C

66. Let G = S_3 be the permutation group of 3 symbols. Then
1. G is isomorphic to a subgroup of a cyclic group
2. there exists a cyclic group H such that G maps homomorphically onto H
3. G is a product of cyclic groups
4. there exists a nontrivial group homomorphism from G to the additive group ($\mathbb{Q}$, +) of rational numbers

67. Let S be the set of polynomials f(x) with integer coefficients satisfying
$f(x) \equiv 1 \bmod (x – 1)$; $f(x) \equiv 0 \bmod (x – 3)$. Which of the following statements are true?
1. S is empty
2. S is a singleton
3. S is a finite non-empty set
4. S is countably infinite

68. Which of the following statements are true?
1. The multiplicative group of a finite field is always cyclic
2. The additive group of a finite field is always cyclic
3. There exists a finite field of any given order
4. There exists at most one finite field (upto isomorphism) of any given order

69. Which of the following statements are true?
1. A subring of an integral domain is an integral domain
2. A subring of a unique factorization domain (U.F.D.) is a U.F.D.
3. A subring of a principal ideal domain (P.I.D.) is a P.I.D.
4. A subring of an Euclidean domain is an Euclidean domain

70. Let G be a group with |G| = 96. Suppose H and K are subgroups of G with |H| = 12 and |K| = 16. Then
1. $H \cap K = \{e\}$
2. $H \cap K \neq \{e\}$
3. $H \cap K$ is Abelian
4. $H \cap K$ is not Abelian

December – 2018

PART - B

71. The number of group homomorphisms from the alternating group A_5 to the symmetric group S_4 is:
1. 1 2. 12 3. 20 4. 6

72. Let p≥23 be a prime number such that the decimal expansion (base 10) of $\frac{1}{p}$ is periodic with period p-1 (that is, $\frac{1}{p} = 0.\overline{a_1 a_2 ... a_{p-1}}$)

with $a_i \in \{0,1,...,9\}$ for all i and for any m,1 ≤ m<p-1, $\frac{1}{p} \neq 0.\overline{a_1 a_2 ... a_m})$. Let (ℤ/pℤ)* denote the multiplicative group of integers modulo p. Then which of the following is correct ?

1. The order of 10∈ (ℤ/pℤ)* is a proper divisor of (p-1)
2. The order of 10 ∈(ℤ/pℤ)* is $\frac{(p-1)}{2}$
3. The element 10∈(ℤ/pℤ)* is a generator of the group (ℤ/pℤ)*
4. The group (ℤ/pℤ)* is cyclic but not generated by the element 10.

73. Given integers a and b, let $N_{a,b}$ denote the number of positive integers k < 100 such that k≡a (mod 9) and k ≡b (mod 11). Then which of the following statements is correct?

1. $N_{a,b}$ = 1 for all integers a and b
2. There exist integers a and b satisfying $N_{a,b}$ > 1.
3. There exist integers a and b satisfying $N_{a,b}$ = 0
4. There exists integers a and b satisfying $N_{a,b}$ = 0 and there exists intgers c and d satisfying $N_{c,d}$ > 1

PART - C

74. For any group G, let Aut(G) denote the group of automorphisms of G. Which of the following are true?

1. If G is finite, then Aut(G) is finite
2. If G is cyclic, then Aut(G) is cyclic
3. If G is infinite, then Aut(G) is infinite
4. If Aut (G) is isomorphic to Aut (H), where G and H are two groups, then G is isomorphic to H

75. Let G be a group with the following property: Given any positive integers m, n and r there exist elements g and h in G such that order(g) = m, order(h) = n and order(gh) = r. Then which of the following are necessarily true?

1. G has to be an infinite group
2. G cannot be a cyclic group
3. G has infinitely many cyclic subgroups
4. G has to be a non-abelian group

76. Let R be the ring ℂ[x]/(x^2 + 1). Pick the correct statements from below:

1. $\dim_{\mathbb{C}}$ R = 3
2. R has exactly two prime ideals
3. R is a UFD
4. (x) is a maximal ideal of R

77. Let f(x) = x^7 – 105x + 12. Then which of the following are correct?

1. f(x) is reducible over ℚ
2. There exists an integer m such that f(m) = 105
3. There exists an integer m such that f(m) = 2
4. f(m) is not a prime number for any integer m

78. Let $\alpha = \sqrt[5]{2} \in$ ℝ and $\xi = \exp\left(\frac{2\pi i}{5}\right)$. Let K = ℚ(αξ). Pick the correct statements from below:

1. There exists a field automorphism σ of ℂ such that σ(K) = K and σ ≠ id
2. There exists a field automorphism σ of ℂ such that σ(K) ≠ K
3. There exists a finite extension E of ℚ such that K ⊆ E and σ(K) ⊆ E for every field automorphism σ of E
4. For all field automorphisms σ of K, σ(αξ) = αξ

JUNE - 2019

PART - B

79. For any integer n ≥ 1, let
d(n) = number of positive divisors of n
v(n) = number of distinct prime divisors of n
w(n) = number of prime divisors of n counted with multiplicity
[for example: If p is prime, then
d(p) = 2, v(p) = v(p^2) = 1, w(p^2) = 2]

1. If n ≥ 1000 and w (n) ≥ 2, then d(n) > log n
2. there exists n such that $d(n) > 3\sqrt{n}$
3. for every n, $2^{v(n)} \leq d(n) \leq 2^{w(n)}$
4. if w(n) = w(m), then d(n) = d(m)

80. Consider the set of matrices

G = $\left\{ \begin{pmatrix} s & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{Z},\ s \in \{-1, +1\} \right\}$

Then which of the following is true?

1. G forms a group under addition

2. G forms an abelian group under multiplication
3. Every element in G is diagonalizable over $\mathbb{C}$
4. G is finitely generated group under multiplication

81. Let R be a commutative ring with unity. Which of the following is true?
1. If R has finitely many prime ideals, then R is a field.
2. If R has finitely many ideals, then R is finite
3. If R is a P.I.D., then every subring of R with unity is a P.I.D.
4. If R is an integral domain which has finitely many ideals, then R is a field.

PART – C

82. Let $a \in \mathbb{Z}$ be such that $a = b^2 + c^2$, where $b, c \in \mathbb{Z}\setminus\{0\}$. Then a cannot be written as
1. pd^2, where $d \in \mathbb{Z}$ and p is a prime with $p \equiv 1 \pmod 4$
2. pd^2, where $d \in \mathbb{Z}$ and p is a prime with $p \equiv 3 \pmod 4$
3. pqd^2, where $d \in \mathbb{Z}$ and p, q are primes with $p \equiv 1 \pmod 4$, $q \equiv 3 \pmod 4$
4. pqd^2, where $d \in \mathbb{Z}$ and p, q are distinct primes with $p, q \equiv 3 \pmod 4$

83. For any prime p, consider the group $G = GL_2(\mathbb{Z}/p\mathbb{Z})$.
Then which of the following are true?
1. G has an element of order p
2. G has exactly one element of order p
3. G has no p-Sylow subgroups
4. Every element of order p is conjugate to a matrix $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, where $a \in (\mathbb{Z}/p\mathbb{Z})^*$

84. Let $\mathbb{Z}[X]$ be the ring of polynomials over integers. Then the additive group $\mathbb{Z}[X]$ is
1. isomorphic to the multiplicative group $\mathbb{Q}^+$ of positive rational numbers
2. isomorphic to the group of rational numbers $\mathbb{Q}$ under addition
3. countable
4. uncountable

85. Let X = (0, 1) be the open unit interval and C (X, $\mathbb{R}$) be the ring of continuous functions from X to $\mathbb{R}$. For any $x \in (0, 1)$, let $I(x) = \{f \in C(X, \mathbb{R}) \mid f(x) = 0\}$. Then which of the following are true?
1. I(x) is a prime ideal
2. I(x) is a maximal ideal
3. Every maximal ideal of C (X, $\mathbb{R}$) is equal to I(x) for some $x \in X$
4. C (X, $\mathbb{R}$) is an integral domain

86. Let $n \in \mathbb{Z}$. Then which of the following are correct?
1. $X^3 + nX + 1$ is irreducible over $\mathbb{Z}$ for every n
2. $X^3 + nX + 1$ is reducible over $\mathbb{Z}$ if $n \in \{0, -2\}$
3. $X^3 + nX + 1$ is irreducible over $\mathbb{Z}$ if $n \notin \{0, -2\}$
4. $X^3 + nX + 1$ is reducible over $\mathbb{Z}$ for infinitely many n

87. Let $\mathbf{F}_{27}$ denote the finite field of size 27. For each $\alpha \in \mathbf{F}_{27}$, we define
$A_\alpha = \{1, 1 + \alpha, 1 + \alpha + \alpha^2, 1 + \alpha + \alpha^2 + \alpha^3, \ldots\}$.
Then which of the following are true?
1. the number of $\alpha \in \mathbf{F}_{27}$ such that $|A_\alpha| = 26$ equals 12
2. $0 \in A_\alpha$ if and only if $\alpha \neq 0$
3. $|A_1| = 27$
4. $\bigcap_{\alpha \in \mathbf{F}_{27}} A_\alpha$ is a singleton set

DECEMBER 2019

PART – B

88. Let G be a group of order p^n, p a prime number and n > 1. Then which of the following is true?
(1) Centre of G has at least two elements
(2) G is always an Abelian group
(3) G has exactly two normal subgroups (i.e., G is a simple group)
(4) If H is any other group of order p^n, then G is isomorphic to H

89. Let S_5 be the symmetric group on five symbols. Then which of the following statements is false?
(1) S_5 contains a cyclic subgroup of order 6
(2) S_5 contains a non-Abelian subgroup of order 8

(3) S_5 does not contain a subgroup isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$
(4) S_5 does not contain a subgroup of order 7

90. A permutation σ of $[n] = \{1, 2, \ldots, n\}$ is called irreducible, if the restriction $\sigma|_{[k]}$ is not a permutation of $[k]$ for any $1 \le k < n$. Let a_n be the number of irreducible permutations of $[n]$. Then $a_1 = 1$, $a_2 = 1$ and $a_3 = 3$. The value of a_4 is
(1) 12 (2) 13
(3) 14 (4) 15

PART – C

91. Let I be an ideal of $\mathbb{Z}$. Then which of the following statements are true?
(1) I is a principal ideal
(2) I is a prime ideal of $\mathbb{Z}$
(3) If I is a prime ideal of $\mathbb{Z}$, then I is a maximal ideal in $\mathbb{Z}$
(4) If I is a maximal ideal in $\mathbb{Z}$, then I is a prime ideal of $\mathbb{Z}$

92. Let $f(x) \in \mathbb{Z}[x]$ be a monic polynomial of degree n. Then which of the following are true?
(1) If f(x) is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{Q}[x]$
(2) If f(x) is irreducible in $\mathbb{Q}[x]$, then it is irreducible in $\mathbb{Z}[x]$
(3) If f(x) is reducible in $\mathbb{Z}[x]$, then it has a real root
(4) If f(x) has a real root, then it is reducible in $\mathbb{Z}[x]$

93. Let F[X] be the polynomial ring in one variable over a field F. Then which of the following statements are true?
(1) F[X] is a UFD
(2) F[X] is a PID
(3) F[X] is a Euclidean domain
(4) F[X] is a PID but is not an Euclidean domain

94. Let C[0, 1] be the ring of all real valued continuous function on [0, 1]
Let
$$A = \left\{ f \in C[0,1] : f\left(\frac{1}{4}\right) = f\left(\frac{3}{4}\right) = 0 \right\}.$$ The n which of the following statements are true?
(1) A is an ideal in C[0, 1] but is not a prime ideal in C[0, 1]
(2) A is a prime ideal in C[0, 1]
(3) A is a maximal ideal in C[0, 1]
(4) A is a prime ideal in C[0, 1], but is not a maximal ideal in C[0, 1]

95. For a given integer k, which of the follow statements are false?
(1) If k (mod 72) is a unit in $\mathbb{Z}_{72}$, then k (mod 9) is a unit in $\mathbb{Z}_9$
(2) If k (mod 72) is a unit in $\mathbb{Z}_{72}$, then k (mod 8) is a unit in $\mathbb{Z}_8$
(3) If k (mod 8) is a unit in $\mathbb{Z}_8$, then k (mod 72) is a unit in $\mathbb{Z}_{72}$
(4) If k (mod 9) is a unit in $\mathbb{Z}_9$, then k (mod 72) is a unit in $\mathbb{Z}_{72}$

96. Let F be a field. Then which of the following statements are true?
(1) All extensions of degree 2 of F are isomorphic as fields
(2) All finite extensions of F of same degree are isomorphic as fields if Char(F) > 0
(3) All finite extensions of F of same degree are isomorphic as fields if F is finite
(4) All finite normal extensions of F are isomorphic as fields if Char(F) = 0

JUNE 2020

PART – B

97. Which of the following statements is true?
(1) Every even integer $n \ge 16$ divides $(n-1)! + 3$
(2) Every odd integer $n \ge 16$ divides $(n-1)!$
(3) Every even integer $n \ge 16$ divides $(n-1)!$
(4) For every integer $n \ge 16$, n^2 divides $n! + 1$

98. Let X be a non-empty set and P(X) be the set of all subsets of X. On P(X), defined two operations $*$ and Δ as follows: for A, B $\in$ P(X), $A * B = A \cap B$; $A \Delta B = (A \cup B) \setminus (A \cap B)$.
Which of the following statements is true?
(1) P(X) is a group under $*$ as well as under Δ

(2) P(X) is a group under $*$, but not under Δ
(3) P(X) is a group under Δ, but not under $*$
(4) P(X) is neither a group under $*$ not under Δ

99. Let $\varphi(n)$ be the cardinality of the set $\{a \mid 1 \le a \le n, (a, n) = 1\}$ where (a, n) denotes the gcd of a and n. Which of the following is NOT true?
(1) There exist infinitely many n such that $\varphi(n) > \varphi(n + 1)$.
(2) There exist infinitely many n such that $\varphi(n) < \varphi(n + 1)$
(3) There exists $N \in \mathbb{N}$ such that $N > 2$ and for all $n > N$, $\varphi(N) < \varphi(n)$
(4) The set $\left\{\frac{\varphi(n)}{n} : n \in \mathbb{N}\right\}$ has finitely many limit points

PART – C

100. Which of the following statements are true?
(1) $\mathbb{Q}$ has countably many subgroups
(2) $\mathbb{Q}$ has uncountably many subsets
(3) Every finitely generated subgroup of $\mathbb{Q}$ is cyclic
(4) $\mathbb{Q}$ is isomorphic to $\mathbb{Q} \times \mathbb{Q}$ as groups

101. Let $SL_2(\mathbb{Z}) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{Z}) : ad - bc = 1\right\}$ and for any prime p, let

$$\Gamma(p) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z}) \;\middle|\; \begin{matrix} a \equiv 1 (\text{mod } p), d \equiv 1 (\text{mod } p) \\ c \equiv 0 (\text{mod } p), b \equiv 0 (\text{mod } p) \end{matrix}\right\}$$

Which of the following are true?
(1) $\Gamma(p)$ is a subgroup of $SL_2(\mathbb{Z})$
(2) $\Gamma(p)$ is not a normal subgroup of $SL_2(\mathbb{Z})$
(3) $\Gamma(p)$ has atleast two elements
(4) $\Gamma(p)$ is uncountable

102. Let G be a finite group. Which of the following are true?
(1) If $g \in G$ has order m and if $n \ge 1$ divides m, then G has a subgroup of order n.
(2) If for any two subgroups A and B of G, either $A \subset B$ or $B \subset A$, then G is cyclic.
(3) If G is cyclic, then for any two subgroups A and B of G, either $A \subset B$ or $B \subset A$
(4) If for every positive integer m dividing $|G|$, G has a subgroup of order m, then G is abelian

103. Let R, S be commutative rings with unity, $f : R \to S$ be a surjective ring homomorphism, $Q \subseteq S$ be a non-zero prime ideal. Which of the following statements are true?
(1) $f^{-1}(Q)$ is a non-zero prime ideal in R
(2) $f^{-1}(Q)$ is a maximal ideal in R if R is a PID
(3) $f^{-1}(Q)$ is a maximal ideal in R if R is a finite commutative ring with unity
(4) $f^{-1}(Q)$ is a maximal ideal in R if $x^5 = x$ for all $x \in R$

104. Consider the polynomial $f(x) = x^2 + 3x - 1$. Which of the following statements are true?
(1) f is irreducible over $\mathbb{Z}[\sqrt{13}]$
(2) f is irreducible over $\mathbb{Q}$
(3) f is reducible over $\mathbb{Q}[\sqrt{13}]$
(4) $\mathbb{Z}[\sqrt{13}]$ is a unique factorization domain

105. Let p be an odd prime such that $p \equiv 2 \pmod 3$. Let $\mathbb{F}_p$ be the field with p elements. Consider the subset E of $\mathbb{F}_p \times \mathbb{F}_p$ given by $E = \{(x, y) \in \mathbb{F}_p \times \mathbb{F}_p : y^2 = x^3 + 1\}$. Which of the follownig are true?
(1) E has alteast two elements
(2) E has atmost 2p elements
(3) E can have p^2 elements
(4) E has alteast 2p elements

JUNE 2021

PART – B

106. Let $S = \{n : 1 \le n \le 999;\ 3|n \text{ or } 37|n\}$. How many integers are there in the set $S^c = \{n : 1 \le n \le 999 ;\ n \notin S\}$?

(1) 639	(2) 648
(3) 666	(4) 990

107. How many generators does a cyclic group of order 36 have?
(1) 6 (2) 12
(3) 18 (4) 24

108. Which of the following statements is necessarily true for a commutative ring R with unity?
(1) R may have no maximal ideals
(2) R can have exactly two maximal ideals
(3) R can have one or more maximal ideals but no prime ideals
(4) R has at least two prime ideals

PART – C

109. A positive integer n co-prime to 17, is called a primitive root modulo 17 if n^k-1 is not divisible by 17 for all k with $1 \le k < 16$. Let a, b be distinct positive integers between 1 and 16. Which of the following statements are true?
(1) 2 is a primitive root modulo 17
(2) If a is a primitive root modulo 17, then a^2 is not necessarily a primitive Root modulo 17
(3) If a, b are primitive roots modulo 17, then ab is a primitive root modulo 17
(4) Product of primitive roots modulo 17 between 1 and 16 is congruent to 1 modulo 17

110. For a positive integer n, let $\Omega(n)$ denote the number of prime factors of n, counted with multiplicity. For instance, $\Omega(3) = 1$, $\Omega(6) = \Omega(9) = 2$. Let $p > 3$ be a prime number and let $N = p(p + 2)(p + 4)$. Which of the following statements are true?
(1) $\Omega(N) \ge 3$
(2) There exist primes $p > 3$ such that $\Omega(N) = 3$
(3) p can never be the smallest prime divisor of N
(4) p can be the smallest prime divisor of N

111. Let G be a group of order 24. Which of the following statements are necessarily true?
(1) G has a normal subgroup of order 3
(2) G is not a simple group
(3) There exists an injective group homomorphism from G to S_8
(4) G has a subgroup of index 4

112. Which of the following statements are true?
(1) All finite field extensions of $\mathbb{Q}$ are Galois
(2) There exists a Galois extension of $\mathbb{Q}$ of degree 3
(3) All finite field extensions of $\mathbb{F}_2$ are Galois
(4) There exists a field extension of $\mathbb{Q}$ of degree 2 which is not Galois

113. Let $f = a_0 + a_1X + \dots a_nX^n$ be a polynomial with $a_i \in \mathbb{Z}$ for $0 \le i \le n$. Let p be a prime such that $p|a_i$ for all $1 < i \le n$ and p^2 does not divide a_n. Which of the following statements are true?
(1) f is always irreducible
(2) f is always reducible
(3) f can sometimes be irreducible and can sometimes be reducible
(4) f can have degree 1

JUNE 2022

PART – B

114. Let R be a ring and N be the set of nilpotent elements i.e. $N = \{x \in R \mid x^n = 0$ for some $n \in \mathbb{N}\}$. Which of the following is true?
(1) N is an ideal in R
(2) N is never an ideal in R
(3) If R is non-commutative, N is not an ideal
(4) If R is commutative, N is an ideal

115. Let R be a commutative ring with identity. Let S be a multiplicatively closed set such that $0 \notin S$. Let I be an ideal which is maximal with respect to the condition that $S \cap I = \varnothing$.
Which of the following is necessarily true?
(1) I is a maximal ideal
(2) I is a prime ideal
(3) I = (1)
(4) I = (0)

116. Let G be a simple group of order 168. How many elements of order 7 does it have
(1) 6 (2) 7
(3) 48 (4) 56

PART – C

117. Let a, b be positive integers with a > b and a + b = 24. Suppose that the following congruences have a common integer solution: $2x \equiv 3a \pmod 5$, $x \equiv 4b \pmod 5$.

Which of the following statements are true?
(1) $10 \leq a - b \leq 20$
(2) $3b > a > 2b$
(3) $a > 3b$
(4) $a - b$ is divisible by 5

118. Consider the function $f(n) = n^5 - 2n^3 + n$, where n is a positive integer. Which of the following statements are true?
(1) For every positive integer k, there exists a positive integer n such that f(n) is divisible by 2^k.
(2) f(n) is even for every integer $n \geq 20$.
(3) For every integer ≥ 20, either f(n) is odd or f(n) divisible by 4.
(4) For every odd integer ≥ 21, f(n) is divisible by 64.

119. Let $A = \mathbb{Z}[X]/(X^2 + X + 1, X^3 + 2X^2 + 2X + 6)$.
Which of the following statements are true?
(1) A is an integral domain
(2) A is a finite ring
(3) A is a field
(4) A is a product of two rings

120. Which of the following statements are necessarily true regarding a group G of order 2022?
(1) Let g be an element of odd order in G and s_g the permutation of G given by $s_g(x) = g(x)$, $x \in G$. Then s_g is even permutation
(2) The set $H = \{g \in G \mid \text{order } (g) \text{ is odd}\}$ is a normal subgroup of G
(3) G has a normal subgroup of index 337
(4) G has only 2 normal subgroups

121. Let p be a prime number and let $\overline{\mathbb{F}_p}$ denote an algebraic closure of the field $\mathbb{F}_p$. We define
$S = \{F \subseteq \overline{\mathbb{F}_p} \mid [F : \mathbb{F}_p] < \infty\}$
Which of the following statements are true?
(1) S is an uncountable set
(2) S is a countable set
(3) For every positive integer n > 1, there exists a unique field $F \in S$ such that $[F : \mathbb{F}_p] = n$
(4) Given any two fields $F_1, F_2 \in S$, either $F_1 \subseteq F_2$ or $F_2 \subseteq F_1$

122. Which of the following are class equations for a finite group?
(1) 1 + 3 + 3 + 3 + 3 + 13 + 13 = 39
(2) 1 + 1 + 2 + 2 + 2 + 2 + 2 + 2 = 14
(3) 1 + 3 + 3 + 7 + 7 = 21
(4) 1 + 1 + 1 + 2 + 5 + 5 = 15

123. Consider $\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{1-\sqrt{5}}{2}$.
Define a sequence of numbers F_n as follows:
$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$ for n = 1, 2, …
Let $p : \mathbb{R} \to \mathbb{R}$ be a polynomial of degree at most 2 such that
$p(1) = F_1$, $p(3) = F_3$, $p(5) = F_5$.
Which of the following statements are true?
(1) $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$
(2) p(7) = 13
(3) $F_n = F_{n-1} + 2F_{n-2}$ for $n \geq 5$
(4) p(7) = 10

JUNE 2023

PART – B

124. Let p be a prime number. Let G be a group such that for each $g \in G$ there exists an $n \in \mathbb{N}$ such that $g^{p^n} = 1$. Which of the following statements if false?
(1) If $|G| = p^6$, then G has a subgroup of index p^2.
(2) If $|G| = p^6$, then G has atleast five normal subgroups.
(3) Center of G can be infinite.
(4) There exists G with $|G| = p^6$ such that G has exactly six normal subgroups.

125. The number of solutions of the equation $x^2 = 1$ in the ring $\mathbb{Z}/105\mathbb{Z}$ is
(1) 0 (2) 2
(3) 4 (4) 8

126. Which of the following equations can occur as the class equation of a group of order 10?
(1) 10 = 1 + 1 + … + 1 (10-times)
(2) 10 = 1 + 1 + 2 + 2 + 2 + 2
(3) 10 = 1 + 1 + 1 + 2 + 5
(4) 10 = 1 + 2 + 3 + 4

PART – C

127. Which of the following statements are correct?

(1) If G is a group of order 244, then G contains a unique subgroup of order 27.
(2) If G is a group of order 1694, then G contains a unique subgroup of order 121.
(3) There exists a group of order 154 which contains a unique subgrouop of order 7.
(4) There exists a group of order 121 which contains two subgroups of order 11.

128. Which of the followng are maximal ideals of $\mathbb{Z}[X]$?
(1) Ideal generated by 2 and $(1 + X^2)$
(2) Ideal generated by 2 and $(1 + X + X^2)$
(3) Ideal generated by 3 and $(1 + X^2)$
(4) Ideal generated by 3 and $(1 + X + X^2)$

129. Let E be a finite algebraic Galois extension of F with Galois group G. Which of the following statements are true?
(1) There is an intermediate field K with $K \neq F$ and $K \neq E$ such that K is a Galois extension of F.
(2) If every proper intermediate field K is a Galois extension of F then G is Abelian.
(3) If E has exactly three intermediate fields including F and E then G is Abelian.
(4) If [E : F] = 99 then every intermediate field is a Galois extension of F.

130. Let $n \geq 1$ be a positive integer and S_n the symmetric group on n symbols. Let $\Delta = \{(g, g): g \in S_n\}$. Which of the following statements are necessarily true?
(1) The map f: $S_n \times S_n \to S_n$ given by f(a, b) = ab is a group homomorphism.
(2) Δ is a subgroup of $S_n \times S_n$.
(3) Δ is a normal subgroup of $S_n \times S_n$.
(4) Δ is a normal subgoup of $S_n \times S_n$, if n is a prime number.

131. Let G_1 and G_2 be two groups and $\varphi: G_1 \to G_2$ be a surjective group homomorphism. Which of the following statements are true?
(1) If G_1 is cyclic then G_2 is cyclic
(2) If G_1 is Abelian then G_2 is Abelian
(3) If H is a subgroup of G_1 then $\varphi(H)$ is a subgroup of G_2
(4) If N is a normal subgroup of G_1 then $\varphi(N)$ is a normal subgroup of G_2.

132. Let G be a grop of order 2023. Which of the following statements are true?
(1) G is an Abelian group.
(2) G is cyclic group.
(3) G is a simple group.
(4) G is not a simple group.

DECEMBER 2023

PART – B

133. Consider the field $\mathbb{C}$ together with the Euclidean topology. Let K be a proper subfield of $\mathbb{C}$ that is not contained in $\mathbb{R}$. Which one of the following statements is necessarily true?
(1) K is dense in $\mathbb{C}$.
(2) K is an algebraic extension of $\mathbb{Q}$.
(3) $\mathbb{C}$ is an algebraic extension of K.
(4) The smallest closed subset of $\mathbb{C}$ containing K is NOT a field.

134. Let G be any finite group. Which one of the following is necessarily true?
(1) G is a union of proper subgroups.
(2) G is a union of proper subgroups if |G| has atleast two distinct prime divisors.
(3) If G is abelian, then G is a union of proper subgroups.
(4) G is a union of proper subgroups if and only if G is not cyclic.

135. Which one of the following is equal to $1^{37} + 2^{37} + \ldots + 88^{37}$ in $\mathbb{Z}/89\mathbb{Z}$?
(1) 88 (2) -88
(3) -2 (4) 0

PART – C

136. Which of the following statements are true?
(1) Let G_1 and G_2 be finite groups such that their orders $|G_1|$ and $|G_2|$ are coprime. Then any homomorphism from G_1 to G_2 is trivial.
(2) Let G be a finite group. Let f : G $\to$ G be a group homomorphism such that f fixes more than half of the elements of G. Then f(x) = x for all $x \in G$.

(3) Let G be a finite group having exactly 3 subgroups. Then G is of order p^2 for some prime p.
(4) Any finite abelian group G has alteast d(|G|) subgroups in G, where d(m) denotes the number of positive divisors of m.

137. Let $n \in \mathbb{Z}$ be such that n is congruent to 1 mod 7 and n is congruent to 4 mod 15. Which of the following statements are true?
(1) n is congruent to 1 mod 3.
(2) n is congruent to 1 mod 35.
(3) n is congruent to 1 mod 21.
(4) n is congruent to 1 mod 5.

138. Let G be the group (under matrix multiplication) of 2×2 invertible matrices with entries from $\mathbb{Z}/9\mathbb{Z}$. Let a be the order of G. Which of the following statements are true?
(1) a is divisible by 3^4.
(2) a is divisible by 2^4.
(3) a is not divisible by 48.
(4) a is divisible by 3^6.

139. Let $R = \mathbb{Z}[X]/(X^2 + 1)$ and $\psi : \mathbb{Z}[X] \to R$ be the natural quotient map. Which of the following statements are true?
(1) R is isomorphic to a subring of $\mathbb{C}$
(2) For any prime number $p \in \mathbb{Z}$, the ideal generated by $\psi(p)$ is a proper ideal of R.
(3) R has infinitely many prime ideals.
(4) The ideal generated by $\psi(X)$ is a prime ideal in R.

140. Let $f(X) = X^2 + X + 1$ and $g(X) = X^2 + X - 2$ be polynomials in $\mathbb{Z}[X]$. Which of the following statements are true?
(1) For all prime numbers p, f(X) mod p is irreducible in $\left(\mathbb{Z}/p\mathbb{Z}\right)[X]$
(2) There exists a prime number p such that g(X) mod p is irreducible in $\left(\mathbb{Z}/p\mathbb{Z}\right)[X]$
(3) g(X) is irreducible in $\mathbb{Q}[X]$
(4) f(X) is irreducible in $\mathbb{Q}[X]$

141. Let $f(X) = X^3 - 2 \in \mathbb{Q}[X]$ and let $K \subset \mathbb{C}$ be the splitting field of f(X) over $\mathbb{Q}$. Let $\omega = e^{2\pi i/3}$. Which of the following statements are true?
(1) The Galois group of K over $\mathbb{Q}$ is the symmetric group S_3.
(2) The Galois group of K over $\mathbb{Q}(\omega)$ is the symmetric group S_3.
(3) The Galois group of K over $\mathbb{Q}$ is $\mathbb{Z}/3\mathbb{Z}$.
(4) The Galois group of K over $\mathbb{Q}(\omega)$ is $\mathbb{Z}/3\mathbb{Z}$.

ANSWERS

1. (2)	2. (2)	3. (3)
4. (1)	5. (4)	6. (2,3)
7. (1,3)	8. (2,3,4)	9. (1,2,3,4)
10. (3)	11. (2)	12. (3)
13. (4)	14. (1,2,3)	15. (2,3)
16. (1,2,3,4)	17. (1,3)	18. (1,3)
19. (2,4)	20. (1,2,4)	21. (3)
22. (3)	23. (4)	24. (2)
25. (1)	26. (3,4)	27. (1,4)
28. (3,4)	29. (2,4)	30. (4)
31. (1,4)	32. (3)	33. (1)
34. (1)	35. (2)	36. (1,4)
37. (1,2,3,4)	38. (1,4)	39. (1,2,4)
40. (1,2)	41. (4)	42. (3)
43. (1,2,3,4)	44. (2)	45. (1,3,4)
46. (1,2,3)	47. (1,2,3,4)	48. (2,4)
49. (4)	50. (1,2,4)	51. (2,4)
52. (1,4)	53. (1,4)	54. (2,4)
55. (1,4)	56. (2)	57. (2)
58. (1)	59. (3)	60. (1,4)
61. (1,2,3,4)	62. (3,4)	63. (1)
64. (3)	65. (4)	66. (2)
67. (1)	68. (1,4)	69. (1)
70. (2,3)	71. (1)	72. (3)
73. (1)	74. (1)	75. (1,2,3,4)
76. (2)	77. (4)	78.
79. (3)	80. (4)	81. (4)
82. (2,3,4)	83. (1,4)	84. (1,3)
85. (1,2,3)	86. (2,3)	87. (1,2,4)
88. (1)	89. (3)	90. (2)
91. (1,4)	92. (1,2)	93. (1,2,3)
94. (1)	95. (1,2)	96. (3)
97. (3)	98. (3)	99. (4)
100. (2,3)	101. (1,3)	102. (1,2)
103. (1,2,3,4)	104. (1,2,3)	105. (1,2)
106. (2)	107. (2)	108. (2)
109. (2,4)	110. (1,3)	111. (2,4)
112. ( )	113. (3)	114. (4)
115. (2)	116. (3)	117. (1,4)
118. (1,2,4)	119. (1,2,3)	120. (1,2)
121. (2,3)	122. (1,3)	123. (1,4)
124. (4)	125. (4)	126. (1)
127. (2,3,4)	128. (2,3)	129. (3,4)
130. (2)	131. (1,2,3,4)	132. (1,4)
133. (1)	134. (4)	135. (4)
136. (1,2,3,4)	137. (1,3)	138. (1,2)
139. (1,2,3)	140. (4)	141. (1,4)