NUMERICAL ANALYSIS (PREVIOUS PAPERS NET)

DECEMBER - 2014

PART - C

1.   Let $f : \mathbb{R} \to \mathbb{R}$ be a smooth function with non-vanishing derivative. The Newton's method for finding a root of $f(x) = 0$ is the same as

1.   fixed point iteration for the map $g(x) = x - f(x)/f'(x)$

2.   Forward Euler method with unit step length for the differential equation $\dfrac{dy}{dx} + \dfrac{f(y)}{f'(y)} = 0$

3.   fixed point iteration for $g(x) = x + f(x)$

4.   fixed point iteration for $g(x) = x - f(x)$

2.   Which of the following approximations for estimating the derivative of a smooth function f at a point x is of order 2 (i.e., the error term is $O(h^2)$)

1. $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

2. $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

3. $f'(x) \approx \dfrac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

4. $f'(x) \approx \dfrac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$

3.   Let $y(t)$ satisfy the differential equation $y' = \lambda y; \ y(0) = 1$. Then the backward Euler method, for $n \geq 1$ and $h > 0$ $\dfrac{y_n - y_{n-1}}{h} = \lambda y_n; \ y_0 = 1$ yields

1. a first order approximation to $e^{\lambda n h}$

2. a polynomial approximation to $e^{\lambda n h}$

3. a rational function approximation to $e^{\lambda n h}$

4. a Chebyshev polynomial approximation to $e^{\lambda n h}$

JUNE - 2015

PART - C

4.   The following numerical integration formula is exact for all polynomials of degree less than or equal to 3

1. Trapezoidal rule

2. Simpson's $\dfrac{1}{3}$rd rule

3. Simpson's $\dfrac{3}{8}$th rule

4. Gauss- Legendre 2 point formula

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DECEMBER - 2015

PART – B

5. Let $f(x) = ax + 100$ for $a \in \mathbb{R}$. Then the iteration $x_{n+1} = f(x_n)$ for $n \geq 0$ and $x_0 = 0$ converges for
1. $a = 5$　　　　　　　　　　　　2. $a = 1$
3. $a = 0.1$　　　　　　　　　　　4. $a = 10$

PART – C

6. The iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n} \right)$, $n \geq 0$ for a given $x_0 \neq 0$ is an instance of
1. fixed point iteration for $f(x) = x^2 - 2$
2. Newton's method for $f(x) = x^2 - 2$
3. fixed point iteration for $f(x) = \dfrac{x^2 + 2}{2x}$
4. Newton's method for $f(x) = x^2 + 2$

7. Let $f(x) = \sqrt{x+3}$ for $x \geq -3$. Consider the iteration $x_{n+1} = f(x_n)$, $x_0 = 0$; $n \geq 0$ The possible limits of the iteration are
1. -1　　　　　　　　　　　　　2. 3
3. 0　　　　　　　　　　　　　　4. $\sqrt{3+\sqrt{3+\sqrt{3+\ldots}}}$

JUNE - 2016

PART – B

8. Let $f(x) = x^2 + 2x + 1$ and the derivative of f at $x = 1$ is approximated by using the central-difference formula $f'(1) \approx \dfrac{f(1+h) - f(1-h)}{2h}$ with $h = \dfrac{1}{2}$. Then the absolute value of the error in the approximation of $f'(1)$ is equal to
1. 1　　　　2. $1/2$　　　　3. 0　　　　4. $1/12$

PART – C

9. Let $H(x)$ be the cubic Hermite interpolation of $f(x) = x^4 + 1$ on the interval $I = [0,1]$ interpolating at $x = 0$ and $x = 1$. Then
1. $\max_{x \in I} |f(x) - H(x)| = \dfrac{1}{16}$.
2. The maximum of $|f(x) - H(x)|$ is attained at $x = \dfrac{1}{2}$.

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3. $\max_{x \in I} |f(x) - H(x)| = \dfrac{1}{21}.$

4. *The maximum of |f(x)–H(x)| is attained at* $x = \dfrac{1}{4}.$

10. *Consider the Runge-Kutta method of the form* $y_{n+1} = y_n + ak_1 + bk_2$
$k_1 = hf(x_n, y_n)$
$k_2 = hf(x_n + \alpha h, y_n + \beta k_1)$ *to approximate the solution of the initial value problem*
$y'(x) = f(x, y(x)), \ y(x_0) = y_0.$ *Which of the following choices of a, b, α and β yield a second order method ?*

1. $a = \dfrac{1}{2}, b = \dfrac{1}{2}, \alpha = 1, \beta = 1$
2. $a = 1, b = 1, \alpha = \dfrac{1}{2}, \beta = \dfrac{1}{2}$

3. $a = \dfrac{1}{4}, b = \dfrac{3}{4}, \alpha = \dfrac{2}{3}, \beta = \dfrac{2}{3}$
4. $a = \dfrac{3}{4}, b = \dfrac{1}{4}, \alpha = 1, \beta = 1$

11. *Let f : [0,3] $\to$ $\mathbb{R}$ be defined by f(x) = |1 - |x − 2||, where, |·| denotes the absolute value. Then for the numerical approximation of* $\displaystyle\int_0^3 f(x)dx,$ *which of the following statements are true?*

1. *The composite trapezoid rule with three equal subintervals is exact.*
2. *The composite midpoint rule with three equal subintervals is exact.*
3. *The composite trapezoid rule with four equal subintervals is exact.*
4. *The composite midpoint rule with four equal subintervals is exact.*

DECEMBER - 2016

PART - B

12. *The values of α and β, such that* $x_{n+1} = \alpha x_n \left(3 - \dfrac{x_n^2}{a} \right) + \beta x_n \left(1 + \dfrac{a}{x_n^2} \right)$ *has 3rd order convergence to* $\sqrt{a},$ *are*

1. $\alpha = \dfrac{3}{8}, \beta = \dfrac{1}{8}.$
2. $\alpha = \dfrac{1}{8}, \beta = \dfrac{3}{8}.$

3. $\alpha = \dfrac{2}{8}, \beta = \dfrac{2}{8}.$
4. $\alpha = \dfrac{1}{4}, \beta = \dfrac{3}{4}.$

PART - C

13. *The order of linear multi step method* $u_{j+1} = (1-a)u_j + au_{j-1} + \dfrac{h}{4}\{(a+3)u'_{j+1} + (3a+1)u'_{j-1}\}$ *for solving* $u' = f(x,u)$ *is*

1. *2 if a = -1*
2. *2 if a = -2*
3. *3 if a = -1*
4. *3 if a = -2*

JUNE - 2017

PART - B

14. The magnitude of the truncation error for the scheme $f'(x) = Af(x) + Bf(x+h) + Cf(x+2h)$ is equal to

1. $h^2 f'''(\xi)$ if $A = -\dfrac{5}{6h}$, $B = \dfrac{3}{2h}$, $C = -\dfrac{2}{3h}$.

2. $h^2 f'''(\xi)$ if $A = \dfrac{5}{6h}$, $B = \dfrac{3}{2h}$, $C = \dfrac{2}{3h}$.

3. $h^2 f''(x)$ if $A = -\dfrac{5}{6h}$, $B = \dfrac{3}{2h}$, $C = -\dfrac{2}{3h}$.

4. $h^2 f''(x)$ if $A = \dfrac{5}{6h}$, $B = \dfrac{3}{2h}$, $C = \dfrac{2}{3h}$.

DECEMBER – 2017

PART - B

15. The iterative method $x_{n+1} = g(x_n)$ for the solution of $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root $x = 2$ if $g(x)$ equals

1. $x^2 - 2$ 2. $(x - 2)^2 - 6$ 3. $1 + \dfrac{2}{x}$ 4. $\dfrac{x^2 + 2}{2x - 1}$

PART - C

16. Consider the linear system $Ax = b$ with $A = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 2 & -2 \\ -3 & -2 & 1 \end{bmatrix}$. Let x_n denote the nth Gauss-Seidel

iteration and $e_n = x_n - x$. Let M be the corresponding matrix such that $e_{n+1} = Me_n$, $n \geq 0$. Which of the following statements are necessarily true?

1.   all eigenvalues of M have absolute value less than 1

2.   there is an eigenvalues of M with absolute value at least 1

3. e_n converges to 0 as $n \rightarrow \infty$ for all $b \in \mathbb{R}^3$ and any e_0

4. e_n does not converge to 0 as $n \rightarrow \infty$ for any $b \in \mathbb{R}^3$ unless $e_0 = 0$

17. For $f \in C[0,1]$ and $n > 1$, let $T(f) = \dfrac{1}{n}\left[\dfrac{1}{2}f(0) + \dfrac{1}{2}f(1) + \sum_{j=1}^{n-1} f\left(\dfrac{j}{n}\right)\right]$ be an approximation of the

integral $I(f) = \int_0^1 f(x)dx$. For which of the following functions f is $T(f) = I(f)$?

1. $1 + sin2\pi nx$ 2. $1 + cos2\pi nx$
3. $sin^2 2\pi nx$ 4. $cos^2 2\pi(n+1)x$

JUNE - 2018

PART – B

18. The values of a,b,c such that $\int_0^h f(x)dx = h\left\{af(0)+bf\left(\dfrac{h}{3}\right)+cf(h)\right\}$ is exact for polynomials f of degree as high as possible are

1. $a = 0, b = \dfrac{3}{4}, c = \dfrac{1}{4}$ 2. $a = \dfrac{3}{4}, b = \dfrac{2}{4}, c = \dfrac{1}{4}$

3. $a = \dfrac{-2}{4}, b = \dfrac{3}{4}, c = \dfrac{1}{4}$ 4. $a = 0, b = \dfrac{1}{4}, c = \dfrac{3}{4}$

PART - C

19. Assume that a non-singular matrix $A = L + D + U$, where L and U are lower and upper triangular matrices respectively with all diagonal entries are zero, and D is a diagonal matrix. Let x^* be the solution of $Ax = b$. Then the Gauss-Seidel iteration method $x^{(k+1)} = Hx^{(k)} + c$, $k = 0, 1, 2, \ldots$ with $||H|| < 1$ converges to x^* provided H is equal to

1. $-D^{-1}(L + U)$ 2. $-(D + L)^{-1}U$
3. $-D(L + U)^{-1}$ 4. $-(L - D)^{-1}U$

20. The forward difference operator is defined as $\Delta U_n = U_{n+1} - U_n$. Then which of the following difference equations has an unbounded general solution?

1. $\Delta^2 U_n - 3\Delta U_n + 2U_n = 0$

2. $\Delta^2 U_n + \Delta U_n + \dfrac{1}{4}U_n = 0$

3. $\Delta^2 U_n - 2\Delta U_n + 2U_n = 0$

4. $\Delta^2 U_{n+1} - \dfrac{1}{3}\Delta^2 U_n = 0$

DECEMBER – 2018

PART – B

21. Let $f(x)$ be a polynomial of unknown degree taking the values

x	0	1	2	3
$f(x)$	2	7	13	16

All the fourth divided differences are -1/6. Then the coefficient of x^3 is

1. 1/3   2. -2/3   3. 16   4. -1

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PART - C

22. *Let f : [0, 1] → [0, 1] be twice continuously differentiable function with a unique fixed point $f(x_*) = x_*$.*
For a given $x_0 \in (0, 1)$ consider the iteration $x_{n+1} = f(x_n)$ for $n \geq 0$.
If $L = max_{x \in [0,1]} |f'(x)|$, then which of the following are true?
1. *If $L < 1$, then x_n converges to x_*.*
2. *x_n converges to x_* provided $L \geq 1$.*
3. *The error $e_n = x_n - x_*$ satisfies $|e_{n+1}| < L|e_n|$.*
4. *If $f'(x_*) = 0$, then $|e_{n+1}| < C |e_n|^2$ for some $C > 0$.*

23. *Let u(x) satisfy the boundary value problem* $(BVP) \begin{cases} u'' + u' = 0, & x \in (0,1) \\ u(0) = 0 \\ u(1) = 1 \end{cases}$

Consider the finite difference approximation to (BVP)

$(BVP)_h \begin{cases} \dfrac{U_{j+1} - 2U_j + U_{j-1}}{h^2} + \dfrac{U_{j+1} - U_{j-1}}{2h} = 0, j = 1,...,N-1 \\ U_0 = 0 \\ U_N = 1 \end{cases}$

Here U_j is an approximation to $u(x_j)$, where $x_j = jh$, $j = 0, ...,N$ is a partition of [0, 1] with $h = 1/N$ for some positive integer N. Then which of the following are true?
1. *There exists a solution to $(BVP)_h$ of the form $U_j = ar^j + b$ for some $a, b \in \mathbb{R}$ with $r \neq 1$ and r satisfying $(2+h)r^2 - 4r + (2-h) = 0$*
2. *$U_j = (r^j - 1) / (r^N - 1)$ where r satisfies $(2 + h) r^2 - 4r + (2 - h) = 0$ and $r \neq 1$*
3. *u is monotonic in x*
4. *U_j is monotonic in j.*

JUNE – 2019

PART – B

24. *Consider solving the following system by Jacobi iteration scheme*
$x + 2my - 2mx = 1$
$nx + y + nz = 2$

$2mx + 2my + z = 1$, where $m, n \in \mathbb{Z}$. With any initial vector, the scheme converges provided m, n, satisfy
1. $m + n = 3$ 2. $m > n$ 3. $m < n$ 4. $m = n$

PART - C

25. *The values of a, b, c so that the truncation error in the formula*
$\int_{-h}^{h} f(x)\, dx = ahf(-h) + bhf(0) + ahf(h) + ch^2 f'(-h) - ch^2 f'(h)$ is minimum, are

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1. $a = \dfrac{7}{15}, b = \dfrac{16}{15}, c = \dfrac{1}{15}$　　　　　2. $a = \dfrac{7}{15}, b = \dfrac{16}{15}, c = \dfrac{-1}{15}$

3. $a = \dfrac{7}{15}, b = \dfrac{-16}{15}, c = \dfrac{1}{15}$　　　　　4. $a = \dfrac{7}{15}, b = \dfrac{-16}{15}, c = \dfrac{-1}{15}$

26. Consider the equation $x^2 + ax + b = 0$ which has two real roots α and β. Then which of the following iteration scheme converges when x_0 is chosen sufficiently close to α?

1. $x_{n+1} = -\dfrac{ax_n + b}{x_n}, if\ |\alpha| > |\beta|$　　　　　2. $x_{n+1} = -\dfrac{x_n^2 + b}{a}, if\ |\alpha| > 1$

3. $x_{n+1} = -\dfrac{b}{x_n + a}, if\ |\alpha| < |\beta|$　　　　　4. $x_{n+1} = -\dfrac{x_n^2 + b}{a}, if\ 2|\alpha| < |\alpha + \beta|$

DECEMBER – 2019

PART – B

27. Let $x = \xi$ be a solution of $x^4 - 3x^2 + x - 10 = 0$. The rate of convergence for the iterative method $x_{n+1} = 10 - x_n^4 + 3x_n^2$ is equal to

1. 1　　　　　2. 2　　　　　3. 3　　　　　4. 4

PART - C

28. Consider the ordinary differential equation (ODE)

$$\begin{cases} y'(x) + y(x) = 0, & x > 0, \\ \qquad\quad y(0) = 1. \end{cases}$$

and the following numerical scheme to solve the ODE

$$\begin{cases} \dfrac{Y_{n+1} - Y_{n-1}}{2h} + Y_{n-1} = 0,\ n \geq 1, \\ Y_0 = 1, Y_1 = 1. \end{cases}$$

If $0 < h < \dfrac{1}{2}$, then which of the following statements are true?

1. $(Y_n) \to \infty$ as $n \to \infty$

2. $(Y_n) \to 0$ as $n \to \infty$

3. (Y_n) is bounded

4. $\displaystyle\max_{nh \in [0,T]} |y(nh) - Y_n| \to \infty\ as\ T \to \infty$

29. The values of α, A, B, C for which the quadrature formula

$$\int_{-1}^{1} (1-x) f(x)\, dx = Af(-\alpha) + Bf(0) + Cf(\alpha)$$

is exact for polynomials of highest possible degree, are

1. $\alpha = \sqrt{\dfrac{3}{5}}, A = \dfrac{5}{9} + \dfrac{\sqrt{5}}{3\sqrt{3}}, B = \dfrac{8}{9}, C = \dfrac{5}{9} - \dfrac{\sqrt{5}}{3\sqrt{3}}$

2. $\alpha = \sqrt{\dfrac{3}{5}}, A = \dfrac{5}{9} - \dfrac{\sqrt{5}}{3\sqrt{3}}, B = \dfrac{8}{9}, C = \dfrac{5}{9} + \dfrac{\sqrt{5}}{3\sqrt{3}}$

3. $\alpha = \sqrt{\dfrac{3}{5}}, A = \dfrac{5}{9}\left(1 - \dfrac{\sqrt{3}}{\sqrt{5}}\right), B = \dfrac{8}{9}, C = \dfrac{5}{9}\left(1 + \dfrac{\sqrt{3}}{\sqrt{5}}\right)$

4. $\alpha = \sqrt{\dfrac{3}{5}}, A = \dfrac{5}{9}\left(1 + \dfrac{\sqrt{3}}{\sqrt{5}}\right), B = \dfrac{8}{9}, C = \dfrac{5}{9}\left(1 - \dfrac{\sqrt{3}}{\sqrt{5}}\right)$

DECEMBER – 2019 (Assam)

PART - B

30. Assume that $a, b \in \mathbb{R}\backslash\{0\}$ and $a^2 \neq b^2$. Suppose that the Gauss-Seidel method is used to solve the system of equations

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then the set of all values of (a, b) such that the method converges for every choice of initial vector is

1. $\{(a, b) \mid a^2 < b^2\}$

2. $\{(a, b) \mid a < |b|\}$

3. $\{(a, b) \mid |b| < |a|\}$

4. $\{(a, b) \mid a^2 + b^2 < 1\}$

PART - C

31. Consider the first order initial value problem $y'(x) = -y(x)$, $x > 0$, $y(0) = 1$ and the corresponding numerical scheme $4\left(\dfrac{y_{n+1} - y_{n-1}}{2h}\right) - 3\left(\dfrac{y_{n+1} - y_n}{h}\right) = -y_n$, with $y_0 = 1$, $y_1 = e^{-h}$, where h is the step size.

Then which of the following statements are true?

1. The order of the scheme is 1

2. The order of the scheme is 2

3. $|y_n - y(nh)| \to \infty$ as $n \to \infty$

4. $|y_n - y(nh)| \to 0$ as $n \to \infty$

32. Consider the integration formula

$$\int_{x_0}^{x_1} f(x)\,dx = \dfrac{h}{2}[f(x_0) + f(x_1)] + ph^2(f'(x_0) - f'(x_1)),$$

where $h = x_1 - x_0$. Then the constant p such that the above formula gives the exact value of the highest degree polynomial and the degree d of the corresponding polynomial are given by

1. $p = \dfrac{1}{6}, d = 4$

2. $p = \dfrac{1}{12}, d = 3$

3. $p = \dfrac{1}{6}, d = 3$

4. $p = \dfrac{1}{12}, d = 4$

JUNE – 2020

PART - B

33. *Let f be an infinitely differentiable real-valued function on a bounded interval I. Take $n \geq 1$ interpolation points $\{x_0, x_1, ..., x_{n-1}\}$. Take n additional interpolation points*

    $x_{n+j} = x_j + \varepsilon, j = 0, 1, ... , n – 1$

    where $\varepsilon > 0$ is such that $\{x_0, x_1, ..., x_{2n-1}\}$ are all distinct.

    Let p_{2n-1} be the Lagrange interpolation polynomial of degree $2n – 1$ with the interpolation points $\{x_0, x_1, ..., x_{2n-1}\}$ for the function f.

    Let q_{2n-1} be the Hermite interpolation polynomial of degree $2n – 1$ with the interpolation points $\{x_0, x_1, ..., x_{n-1}\}$ for the function f. In the $\varepsilon \to 0$ limit, the quantity

    $$\sup_{x \in I} | p_{2n-1}(x) - q_{2n-1}(x) |$$

    1. does not necessarily converge

    2. converges to $\dfrac{1}{2n}$

    3. converges to 0

    4. converges to $\dfrac{1}{2n+1}$

PART – C

34. *Fix a $\alpha \in (0, 1)$. Consider the iteration defined by*

    (*) $x_{k+1} = \dfrac{1}{2}(x_k^2 + \alpha), \ k = 0, 1, 2, ...$

    The above iteration has two distinct fixed points ζ_1 and ζ_2 such that $0 < \zeta_1 < 1 < \zeta_2$.
    Which of the following statements are true?

    1. *The iteration (*) is equivalent to the recurrence relation $x_{k+2} - \zeta_1 = \dfrac{1}{2}(x_k + \zeta_1)(x_k - \zeta_1), \ k = 0, 1, 2, ...$*

    2. *The iteration (*) is equivalent to the recurrence relation $x_{k+1} - \zeta_1 = \dfrac{1}{2}(x_k + \zeta_2)(x_k - \zeta_1), \ k = 0, 1, 2, ...$*

    3. *If $0 \leq x_0 < \zeta_2$ then $\lim\limits_{k \to \infty} x_k = \zeta_1$*

    4. *If $-\zeta_2 < x_0 \leq 0$ then $\lim\limits_{k \to \infty} x_k = \zeta_1$*

35. *Consider the function $f : [0, 1] \to \mathbb{R}$ defined by*

    $$f(x) := \begin{cases} 2^{-\left\{1+\left(\log_2\left(\frac{1}{x}\right)\right)^{\frac{1}{\beta}}\right\}^{\beta}} & \text{for } x \in (0,1] \\ 0 & \text{for } x = 0, \end{cases}$$

    where $\beta \in (0, \infty)$ is a parameter. Consider the iterations

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$x_{k+1} = f(x_k)$, $k = 0, 1, \dots$; $x_0 > 0$.

Which of the following statements are true about the iteration?

1. For $\beta = 1$, the sequence $\{x_k\}$ converges to 0 linearly with asymptotic rate of convergence $\log_{10} 2$

2. For $\beta > 1$, the sequence $\{x_k\}$ does not converge to 0

3. For $\beta \in (0, 1)$, the sequence $\{x_k\}$ converges to 0 sublinearly

4. For $\beta \in (0, 1)$, the sequence $\{x_k\}$ converges to 0 superlinearly

JUNE – 2020 (Tamil Nadu)

PART - B

36. *Consider the Newton-Raphson method applied to approximate the square root of a positive number α.*

A recursion relation for the error $e_n = x_n - \sqrt{\alpha}$ is given by

1. $e_{n+1} = \dfrac{1}{2}\left(e_n + \dfrac{\alpha}{e_n}\right)$ 2. $e_{n+1} = \dfrac{1}{2}\left(e_n - \dfrac{\alpha}{e_n}\right)$

3. $e_{n+1} = \dfrac{1}{2}\dfrac{e_n^2}{e_n + \sqrt{\alpha}}$ 4. $e_{n+1} = \dfrac{e_n^2}{e_n + 2\sqrt{\alpha}}$

PART – C

37. *Consider the numerical integration formula*

$\displaystyle\int_{-1}^{1} g(x)\,dx \approx g(\alpha) + g(-\alpha),\ where\ \alpha = (0.2)^{1/4}$. *Which of the following statements are true?*

1. *The integration formula is exact for polynomialss of the form $a + bx$, for all $a, b \in \mathbb{R}$*

2. *The integration formula is exact for polynomials of the form $a + bx + cx^2$, for all $a, b, c \in \mathbb{R}$*

3. *The integration formula is exact for polynomials of the form $a + bx + cx^2 + dx^3$, for all $a, b, c, d \in \mathbb{R}$*

4. *The integration formula is exact for polynomials of the form $a + bx + cx^3 + dx^4$ for all $a, b, c, d \in \mathbb{R}$*

JUNE – 2021

PART – B

38. *Let the solution to the initial value problem*

$y' = y - t^2 + 1$, $0 \le t \le 2$, $y(0) = 0.5$

be computed using the Euler's method with step-length $h = 0.4$. If $y(0.8)$ and $w(0.8)$ denote the exact and approximate solutions at $t = 0.8$, then an error bound for Euler's method is given by

1. $0.2(0.5e^2 - 2)(e^{0.4} - 1)$ 2. $0.1(e^{0.4} - 1)$

3. $0.2(0.5e^2 - 2)(e^{0.8} - 1)$ 4. $0.1(e^{0.8} - 1)$

39. Let a, b, $c \in \mathbb{R}$ be such that the quadrature rule

$$\int_{-1}^{1} f(x)\,dx = af(-1) + bf'(0) + cf'(1)$$

is exact for all polynomials of degree less than or equal to 2. The $a + b + c$ equal to

1. 4　　　　　　　　2. 3　　　　　　　　3. 2　　　　　　　　4. 1

PART – C

40. The values of a, b, c, d, e for which the function

$$f(x) = \begin{cases} a(x-1)^2 + b(x-2)^3 & -\infty < x \leq 2 \\ c(x-1)^2 + d & 2 \leq x \leq 3 \\ (x-1)^2 + e(x-3)^3 & 3 \leq x < \infty \end{cases}$$

is a cubic spline are

1. $a = c = 1$, $d = 0$, b, e are arbitrary
2. $a = b = c = 1$, $d = 0$, e is arbitrary
3. $a = b = c = d = 1$, e is arbitrary
4. $a = b = c = d = e = 1$

41. Consider the Euler method for integration of the system of differential equations

$$\dot{x} = -y$$
$$\dot{y} = x$$

Assume that (x_i^n, y_i^n) are the points obtained for $i = 0, 1, ..., n^2$ using a time-step $h = 1/n$ starting at the initial point $(x_0, y_0) = (1, 0)$. Which of the following statements are true?

1. The points (x_i^n, y_i^n) lie on a circle of radius 1
2. $\lim_{n \to \infty}(x_n^n, y_n^n) = (\cos(1), \sin(1))$
3. $\lim_{n \to \infty}(x_2^n, y_2^n) = (1, 0)$
4. $(x_i^n)^2 + (y_i^n)^2 > 1$ for $i \geq 1$

JUNE – 2022

PART - B

42. Let A be following invertible matrix with real positive entries $A = \begin{pmatrix} 1 & 2 \\ 8 & 9 \end{pmatrix}$. Let G be the associated Gauss-Seidel iteration matrix. What are the two eigenvalues of G?

1. 0 and $\dfrac{4}{3}$　　　　　　　　　　　　　　2. 0 and $-\dfrac{4}{3}$

3. 0 and $\dfrac{16}{9}$　　　　　　　　　　　　　　4. $\dfrac{4}{3}$ and $-\dfrac{4}{3}$

PART – C

43. *Consider the ODE $\dot{x} = f(t, x)$ in $\mathbb{R}$, for a smooth function f. Consider a general second order Runge-Kutta formula of the form $x(t + h) = x(t) + w_1 hf(t, x) + w_2 hf(t + \alpha h, x + \beta hf) + 0(h^3)$. Which of the following choices of $(w_1, w_2, \alpha, \beta)$ are correct?*

1. $\left(\dfrac{1}{2}, \dfrac{1}{2}, 1, 1\right)$

2. $\left(\dfrac{1}{2}, 1, \dfrac{1}{2}, 1\right)$

3. $\left(\dfrac{1}{4}, \dfrac{3}{4}, \dfrac{2}{3}, \dfrac{2}{3}\right)$

4. (0, 1, 1, 1)

JUNE – 2023

PART – B

44. *Which of the following values of a, b, c and d will produce a quadrature formula*

$$\int_{-1}^{1} f(x)\, dx \approx af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3?

1. $a = 1, b = 1, c = \dfrac{1}{3}, d = -\dfrac{1}{3}$

2. $a = -1, b = 1, c = \dfrac{1}{3}, d = -\dfrac{1}{3}$

3. $a = 1, b = 1, c = -\dfrac{1}{3}, d = -\dfrac{1}{3}$

4. $a = 1, b = -1, c = \dfrac{1}{3}, d = -\dfrac{1}{3}$

DECEMBER – 2023

PART – B

45. *Using Euler's method with the step size 0.05, the approximate value of the solution for the initial value problem*

$$\frac{dy}{dx} = \sqrt{3x + 2y + 1},\ y(1) = 1,$$

at x = 1.1 (rounded off to two decimal places), is

(1) 1.50

(2) 1.65

(3) 1.25

(4) 1.15

PART – C

46. *The coefficient of x^3 in the interpolating polynomial for the data*

x	0	1	2	3	4
y	1	2	1	3	5

is

(1) $-\dfrac{1}{3}$ 　　　　(2) $-\dfrac{1}{2}$ 　　　　(3) $\dfrac{5}{6}$ 　　　　(4) $\dfrac{17}{6}$

47. *Consider the initial value problem*

$$\dfrac{dy}{dx} = f(x, y),\ y(x_0) = y_0,$$

where f is a twice continuously differentiable function on a rectangle containing the point (x_0, y_0). With the step-size h, let the first iterate of a second order scheme to approximate the solution of the above initial value problem be given by $y_1 = y_0 + Pk_1 + Qk_2$,

where $k_1 = hf(x_0, y_0)$, $k_2 = hf(x_0 + \alpha_0 h, y_0 + \beta_0 k_1)$ and $P, Q, \alpha_0, \beta_0 \in \mathbb{R}$.

Which of the following statements are correct?

(1) If $\alpha_0 = 2$, then $\beta_0 = 2, P = \dfrac{3}{4}, Q = \dfrac{1}{4}$

(2) If $\beta_0 = 3$, then $\alpha_0 = 3, P = \dfrac{5}{6}, Q = \dfrac{1}{6}$

(3) If $\alpha_0 = 2$, then $\beta_0 = 2, P = \dfrac{1}{4}, Q = \dfrac{3}{4}$

(4) If $\beta_0 = 3$, then $\alpha_0 = 3, P = \dfrac{1}{6}, Q = \dfrac{5}{6}$

ANSWER KEY

1. (1,2)	2.(2,3,4)	3. (1, 3)	4. (2,3,4)	5. (3)	6. (2, 3)
7.(4)	8. (3)	9. (1,2)	10.(1,3)	11. (1,2)	12. (2)
13. (2,3)	14. (*)	15.(4)	16. (2,4)	17. (1)	18.(1)
19.(2)	20. (1,3,4)	21. (1)	22. (1,3,4)	23. (1, 2, 3, 4)	24. (4)
25. (1)	26. (1,3,4)	27. (1)	28. (2,3)	29. (1,4)	30. (3)
31. (1,3)	32. (2)	33. (3)	34. (1)	35. (1,3)	36. (3)
37. (1)	38. (3)	39.	40. (1,2)	41. (2,3,4)	42. (3)
43. (1,3)	44. (1)	45.	46.	47.	

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