Joint Admission Test for Masters 2021 14th Feb S2

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Subject	MATHEMATICS

Section : **Section A**

Q.1 Which one of the following subsets of $\mathbb{R}$ has a non-empty interior?

Options

1. The set $\{b \in \mathbb{R} : x^2 + bx + 1 = 0$ has distinct roots$\}$.
2. The set of all irrational numbers in $\mathbb{R}$.
3. The set of all rational numbers in $\mathbb{R}$.
4. The set $\{a \in \mathbb{R} : \sin(a) = 1\}$.

Question Type : **MCQ**
Question ID : **111686308**
Status : **Answered**
Chosen Option : **1**

Q.2 Let $P : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $P(x) > 0$ for all $x \in \mathbb{R}$. Let y be a twice differentiable function on $\mathbb{R}$ satisfying $y''(x) + P(x)y'(x) - y(x) = 0$ for all $x \in \mathbb{R}$. Suppose that there exist two real numbers a, b $(a < b)$ such that $y(a) = y(b) = 0$. Then

Options

1. $y(x) < 0$ for all $x \in (a, b)$.
2. $y(x) = 0$ for all $x \in [a, b]$.
3. $y(x) > 0$ for all $x \in (a, b)$.
4. $y(x)$ changes sign on (a, b).

Question Type : **MCQ**
Question ID : **111686302**
Status : **Not Answered**
Chosen Option : **--**

Q.3 Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $f(x) = f(x+1)$ for all $x \in \mathbb{R}$. Then

Options
1. f is not necessarily bounded above.
2. there is no $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$.
3. there exist infinitely many $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$.
4. there exists a unique $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$.

Question Type : **MCQ**
Question ID : **111686303**
Status : **Answered**
Chosen Option : **3**

Q.4 Let $n > 1$ be an integer. Consider the following two statements for an arbitrary $n \times n$ matrix A with complex entries.

I. If $A^k = I_n$ for some integer $k \geq 1$, then all the eigenvalues of A are k^{th} roots of unity.

II. If, for some integer $k \geq 1$, all the eigenvalues of A are k^{th} roots of unity, then $A^k = I_n$.

Then

Options
1. I is FALSE but II is TRUE.
2. both I and II are TRUE.
3. neither I nor II is TRUE.
4. I is TRUE but II is FALSE.

Question Type : **MCQ**
Question ID : **111686310**
Status : **Answered**
Chosen Option : **1**

Q.5 Let $0 < \alpha < 1$ be a real number. The number of differentiable functions $y : [0,1] \to [0,\infty)$, having continuous derivative on $[0,1]$ and satisfying

$$\begin{aligned} y'(t) &= (y(t))^\alpha, \quad t \in [0,1], \\ y(0) &= 0, \end{aligned}$$

is

Options

1. exactly one.
2. finite but more than two.
3. exactly two.
4. infinite.

Question Type : **MCQ**
Question ID : **111686301**
Status : **Answered**
Chosen Option : **1**

Q.6 Let p and t be positive real numbers. Let D_t be the closed disc of radius t centered at $(0,0)$, i.e., $D_t = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq t^2\}$. Define

$$I(p,t) = \iint_{D_t} \frac{dx dy}{(p^2 + x^2 + y^2)^p}.$$

Then $\lim_{t\to\infty} I(p,t)$ is finite

Options

1. only if $p > 1$.
2. only if $p < 1$.
3. only if $p = 1$.
4. for no value of p.

Question Type : **MCQ**
Question ID : **111686305**
Status : **Answered**
Chosen Option : **1**

Q.7 For every $n \in \mathbb{N}$, let $f_n : \mathbb{R} \to \mathbb{R}$ be a function. From the given choices, pick the statement that is the negation of

"For every $x \in \mathbb{R}$ and for every real number $\epsilon > 0$, there exists an integer $N > 0$ such that $\sum_{i=1}^{p} |f_{N+i}(x)| < \epsilon$ for every integer $p > 0$."

Options

1. For every $x \in \mathbb{R}$ and for every real number $\epsilon > 0$, there does not exist any integer $N > 0$ such that $\sum_{i=1}^{p} |f_{N+i}(x)| < \epsilon$ for every integer $p > 0$.

2. For every $x \in \mathbb{R}$ and for every real number $\epsilon > 0$, there exists an integer $N > 0$ such that $\sum_{i=1}^{p} |f_{N+i}(x)| \geq \epsilon$ for some integer $p > 0$.

3. There exists $x \in \mathbb{R}$ and there exists a real number $\epsilon > 0$ such that for every integer $N > 0$ and for every integer $p > 0$ the inequality $\sum_{i=1}^{p} |f_{N+i}(x)| \geq \epsilon$ holds.

4. There exists $x \in \mathbb{R}$ and there exists a real number $\epsilon > 0$ such that for every integer $N > 0$, there exists an integer $p > 0$ for which the inequality $\sum_{i=1}^{p} |f_{N+i}(x)| \geq \epsilon$ holds.

Question Type : **MCQ**
Question ID : **111686307**
Status : **Not Answered**
Chosen Option : **--**

Q.8 For an integer $k \geq 0$, let P_k denote the vector space of all real polynomials in one variable of degree less than or equal to k. Define a linear transformation $T : P_2 \longrightarrow P_3$ by

$$Tf(x) = f''(x) + xf(x).$$

Which one of the following polynomials is not in the range of T?

Options

1. $x^2 + x^3 + 2$

2. $x + x^3 + 2$

3. $x + 1$

4. $x + x^2$

Question Type : **MCQ**
Question ID : **111686309**
Status : **Answered**
Chosen Option : **2**

Q.9

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that for all $x \in \mathbb{R}$,

$$\int_0^1 f(xt)\, dt = 0. \qquad (*)$$

Then

Options 1.

there is an f satisfying $(*)$ that takes both positive and negative values.

2. f must be identically 0 on the whole of $\mathbb{R}$.

3.

there is an f satisfying $(*)$ that is 0 at infinitely many points, but is not identically zero.

4.

there is an f satisfying $(*)$ that is identically 0 on $(0, 1)$ but not identically 0 on the whole of $\mathbb{R}$.

Question Type : **MCQ**
Question ID : **111686304**
Status : **Not Answered**
Chosen Option : **--**

Q.10

How many elements of the group $\mathbb{Z}_{50}$ have order 10?

Options

1. 8
2. 10
3. 4
4. 5

Question Type : **MCQ**
Question ID : **111686306**
Status : **Answered**
Chosen Option : **3**

Q.11 Let G be a finite abelian group of odd order. Consider the following two statements:

I. The map $f : G \to G$ defined by $f(g) = g^2$ is a group isomorphism.

II. The product $\prod_{g \in G} g = e$.

Options

1. Both I and II are TRUE.
2. II is TRUE but I is FALSE.
3. Neither I nor II is TRUE.
4. I is TRUE but II is FALSE.

Question Type : **MCQ**
Question ID : **111686328**
Status : **Answered**
Chosen Option : **1**

Q.12 Let $f : \mathbb{N} \to \mathbb{N}$ be a bijective map such that

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^2} < +\infty.$$

The number of such bijective maps is

Options

1. finite but more than one.
2. zero.
3. exactly one.
4. infinite.

Question Type : **MCQ**
Question ID : **111686315**
Status : **Answered**
Chosen Option : **2**

Q.13 Let y be the solution of

$$(1+x)y''(x) + y'(x) - \frac{1}{1+x}y(x) = 0, \quad x \in (-1, \infty),$$
$$y(0) = 1, \ y'(0) = 0.$$

Then

Options

1. y attains its minimum at $x = 0$.
2. y is bounded on $(0, \infty)$.
3. y is bounded on $(-1, 0]$.
4. $y(x) \geq 2$ on $(-1, \infty)$.

Question Type : **MCQ**
Question ID : **111686312**
Status : **Not Answered**
Chosen Option : **--**

Q.14 Let $n \geq 2$ be an integer. Let $A : \mathbb{C}^n \longrightarrow \mathbb{C}^n$ be the linear transformation defined by

$$A(z_1, z_2, \ldots, z_n) = (z_n, z_1, z_2, \ldots, z_{n-1}).$$

Which one of the following statements is true for every $n \geq 2$?

Options

1. A is singular.
2. A is nilpotent.
3. All eigenvalues of A are of modulus 1.
4. Every eigenvalue of A is either 0 or 1.

Question Type : **MCQ**
Question ID : **111686329**
Status : **Answered**
Chosen Option : **4**

Q.15 Let $f : [0,1] \to [0,\infty)$ be a continuous function such that

$$(f(t))^2 < 1 + 2\int_0^t f(s)\,ds, \text{ for all } t \in [0,1].$$

Then

Options
1. $f(t) < 1 + \frac{t}{2}$ for all $t \in [0,1]$.
2. $f(t) < 1 + t$ for all $t \in [0,1]$.
3. $f(t) > 1 + t$ for all $t \in [0,1]$.
4. $f(t) = 1 + t$ for all $t \in [0,1]$.

Question Type : **MCQ**
Question ID : **111686321**
Status : **Not Answered**
Chosen Option : **--**

Q.16 Let $M_n(\mathbb{R})$ be the real vector space of all $n \times n$ matrices with real entries, $n \geq 2$. Let $A \in M_n(\mathbb{R})$. Consider the subspace W of $M_n(\mathbb{R})$ spanned by $\{I_n, A, A^2, \ldots\}$. Then the dimension of W over $\mathbb{R}$ is necessarily

Options
1. n^2.
2. ∞.
3. n.
4. at most n.

Question Type : **MCQ**
Question ID : **111686311**
Status : **Answered**
Chosen Option : **2**

Q.17 Consider the family of curves $x^2 - y^2 = ky$ with parameter $k \in \mathbb{R}$. The equation of the orthogonal trajectory to this family passing through $(1,1)$ is given by

Options
1. $x^3 + 3xy^2 = 4$.
2. $y^2 + 2x^2y = 3$.
3. $x^3 + 2xy^2 = 3$.
4. $x^2 + 2xy = 3$.

Question Type : **MCQ**
Question ID : **111686319**
Status : **Answered**
Chosen Option : **1**

Q.18 Let $f : \mathbb{R} \to \mathbb{R}$ be an infinitely differentiable function such that for all $a, b \in \mathbb{R}$ with $a < b$,

$$\frac{f(b) - f(a)}{b - a} = f'\Big(\frac{a+b}{2}\Big).$$

Then

Options

1. f must be a linear polynomial.
2. f is not a polynomial.
3. f must be a polynomial of degree less than or equal to 2.
4. f must be a polynomial of degree greater than 2.

Question Type : **MCQ**
Question ID : **111686317**
Status : **Not Answered**
Chosen Option : **--**

Q.19 Let y be a twice differentiable function on $\mathbb{R}$ satisfying

$$y''(x) = 2 + e^{-|x|}, \quad x \in \mathbb{R},$$
$$y(0) = -1, \quad y'(0) = 0.$$

Then

Options

1. there exists an $x_0 \in \mathbb{R}$ such that $y(x_0) \geq y(x)$ for all $x \in \mathbb{R}$.
2. $y = 0$ has exactly one root.
3. $y = 0$ has more than two roots.
4. $y = 0$ has exactly two roots.

Question Type : **MCQ**
Question ID : **111686325**
Status : **Not Answered**
Chosen Option : **--**

Q.20 Define

$$S = \lim_{n\to\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right).$$

Then

Options

1. $S = 1/2.$
2. $S = 1.$
3. $S = 3/4.$
4. $S = 1/4.$

Question Type : **MCQ**
Question ID : **111686316**
Status : **Answered**
Chosen Option : **1**

Q.21 Which one of the following statements is true?

Options

1. $(\mathbb{Q}/\mathbb{Z}, +)$ is isomorphic to $(\mathbb{Q}/2\mathbb{Z}, +)$.
2. $(\mathbb{Z}, +)$ is isomorphic to $(\mathbb{R}, +)$.
3. $(\mathbb{Q}/\mathbb{Z}, +)$ is isomorphic to $(\mathbb{Q}, +)$.
4. $(\mathbb{Z}, +)$ is isomorphic to $(\mathbb{Q}, +)$.

Question Type : **MCQ**
Question ID : **111686324**
Status : **Answered**
Chosen Option : **1**

Q.22 Consider the following statements.

I. The group $(\mathbb{Q}, +)$ has no proper subgroup of finite index.

II. The group $(\mathbb{C} \setminus \{0\}, \cdot)$ has no proper subgroup of finite index.

Which one of the following statements is true?

Options

1. I is TRUE but II is FALSE.
2. Both I and II are TRUE.
3. Neither I nor II is TRUE.
4. II is TRUE but I is FALSE.

Question Type :	**MCQ**
Question ID :	**111686314**
Status :	**Not Attempted and Marked For Review**
Chosen Option :	**--**

Q.23 Let g be an element of S_7 such that g commutes with the element $(2, 6, 4, 3)$. The number of such g is

Options

1. 24.
2. 6.
3. 48.
4. 4.

Question Type :	**MCQ**
Question ID :	**111686327**
Status :	**Answered**
Chosen Option :	**1**

Q.24

Let A be an $n \times n$ invertible matrix and C be an $n \times n$ nilpotent matrix. If $X = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}$ is a $2n \times 2n$ matrix (each X_{ij} being $n \times n$) that commutes with the $2n \times 2n$ matrix $B = \begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}$, then

Options

1. X_{11} and X_{22} are necessarily zero matrices.
2. X_{12} and X_{21} are necessarily zero matrices.
3. X_{12} and X_{22} are necessarily zero matrices.
4. X_{11} and X_{21} are necessarily zero matrices.

Question Type : **MCQ**
Question ID : **111686322**
Status : **Not Answered**
Chosen Option : **--**

Q.25

Consider the surface $S = \{(x, y, xy) \in \mathbb{R}^3 : x^2 + y^2 \leq 1\}$. Let $\vec{F} = y\hat{i} + x\hat{j} + \hat{k}$. If $\hat{n}$ is the continuous unit normal field to the surface S with positive z-component, then

$$\iint_S \vec{F} \cdot \hat{n}\, dS$$

equals

Options

1. π.
2. 2π.
3. $\frac{\pi}{4}$.
4. $\frac{\pi}{2}$.

Question Type : **MCQ**
Question ID : **111686313**
Status : **Answered**
Chosen Option : **4**

Q.26 Which one of the following statements is true?

Options
1. Any abelian subgroup of S_5 is trivial.
2. Exactly half of the elements in any even order subgroup of S_5 must be even permutations.
3. There exists a normal subgroup of S_5 of index 7.
4. There exists a cyclic subgroup of S_5 of order 6.

Question Type : **MCQ**
Question ID : **111686320**
Status : **Answered**
Chosen Option : **4**

Q.27 Consider the two series

$$\text{I.} \quad \sum_{n=1}^{\infty} \frac{1}{n^{1+(1/n)}} \quad \text{and} \quad \text{II.} \quad \sum_{n=1}^{\infty} \frac{1}{n^{2-n^{1/n}}}.$$

Which one of the following holds?

Options
1. I converges and II diverges.
2. Both I and II converge.
3. Both I and II diverge.
4. I diverges and II converges.

Question Type : **MCQ**
Question ID : **111686330**
Status : **Answered**
Chosen Option : **4**

Q.28 Let $D \subseteq \mathbb{R}^2$ be defined by $D = \mathbb{R}^2 \setminus \{(x, 0) : x \in \mathbb{R}\}$. Consider the function $f : D \to \mathbb{R}$ defined by

$$f(x, y) = x \sin \frac{1}{y}.$$

Then

Options
1. f is a continuous function on D and cannot be extended continuously to any point outside D.
2. f is a continuous function on D and can be extended continuously to the whole of $\mathbb{R}^2$.
3. f is a continuous function on D and can be extended continuously to $D \cup \{(0, 0)\}$.
4. f is a discontinuous function on D.

Question Type : **MCQ**
Question ID : **111686323**
Status : **Not Answered**
Chosen Option : **--**

Q.29 Let $f : [0, 1] \to [0, 1]$ be a non-constant continuous function such that $f \circ f = f$. Define

$$E_f = \{x \in [0, 1] : f(x) = x\}.$$

Then

Options
1. E_f need not be an interval.
2. E_f is an interval.
3. E_f is neither open nor closed.
4. E_f is empty.

Question Type : **MCQ**
Question ID : **111686326**
Status : **Answered**
Chosen Option : **2**

Q.30 Consider the function

$$f(x) = \begin{cases} 1 & \text{if } x \in (\mathbb{R} \setminus \mathbb{Q}) \cup \{0\}, \\ 1 - \frac{1}{p} & \text{if } x = \frac{n}{p},\ n \in \mathbb{Z} \setminus \{0\},\ p \in \mathbb{N} \text{ and } \gcd(n, p) = 1. \end{cases}$$

Then

Options

1. f is continuous at all $x \in \mathbb{Q}$.
2. all $x \in \mathbb{Q} \setminus \{0\}$ are strict local minima for f.
3. f is not continuous at all $x \in \mathbb{R} \setminus \mathbb{Q}$.
4. f is not continuous at $x = 0$.

Question Type : **MCQ**
Question ID : **111686318**
Status : **Answered**
Chosen Option : **2**

Section : **Section B**

Q.1 Consider the two functions $f(x, y) = x + y$ and $g(x, y) = xy - 16$ defined on $\mathbb{R}^2$. Then

Options

1. the function g has no global extreme value subject to the condition $f = 0$.
2. the function f attains global extreme values at $(4, 4)$ and $(-4, -4)$ subject to the condition $g = 0$.
3. the function g has a global extreme value at $(0, 0)$ subject to the condition $f = 0$.
4. the function f has no global extreme value subject to the condition $g = 0$.

Question Type : **MSQ**
Question ID : **111686334**
Status : **Answered**
Chosen Option : **2,3**

Q.2 Let V be a finite dimensional vector space and $T : V \to V$ be a linear transformation. Let $\mathcal{R}(T)$ denote the range of T and $\mathcal{N}(T)$ denote the null space $\{v \in V : Tv = 0\}$ of T. If $\text{rank}(T) = \text{rank}(T^2)$, then which of the following is/are necessarily true?

Options

1. $\mathcal{N}(T) = \mathcal{N}(T^2)$.
2. $\mathcal{N}(T) = \{0\}$.
3. $\mathcal{R}(T) = \mathcal{R}(T^2)$.
4. $\mathcal{N}(T) \cap \mathcal{R}(T) = \{0\}$.

Question Type : **MSQ**
Question ID : **111686339**
Status : **Answered**
Chosen Option : **1,4**

Q.3 Consider the four functions from $\mathbb{R}$ to $\mathbb{R}$:

$$f_1(x) = x^4 + 3x^3 + 7x + 1, \quad f_2(x) = x^3 + 3x^2 + 4x, \quad f_3(x) = \arctan(x)$$

and

$$f_4(x) = \begin{cases} x & \text{if } x \notin \mathbb{Z}, \\ 0 & \text{if } x \in \mathbb{Z}. \end{cases}$$

Which of the following subsets of $\mathbb{R}$ are open?

Options

1. The range of f_4.
2. The range of f_2.
3. The range of f_3.
4. The range of f_1.

Question Type : **MSQ**
Question ID : **111686338**
Status : **Answered**
Chosen Option : **3**

Q.4 Consider the equation

$$x^{2021} + x^{2020} + \cdots + x - 1 = 0.$$

Then

Options

1. exactly one real root is negative.
2. exactly one real root is positive.
3. all real roots are positive.
4. no real root is positive.

Question Type : **MSQ**
Question ID : **111686332**
Status : **Answered**
Chosen Option : **2,3**

Q.5 Let $f : (a, b) \to \mathbb{R}$ be a differentiable function on (a, b). Which of the following statements is/are true?

Options

1. If $f'(x_0) > 0$ for some $x_0 \in (a, b)$, then f is increasing in a neighbourhood of x_0.
2. $f' > 0$ in (a, b) implies that f is increasing in (a, b).
3. f is increasing in (a, b) implies that $f' > 0$ in (a, b).
4. If $f'(x_0) > 0$ for some $x_0 \in (a, b)$, then there exists a $\delta > 0$ such that $f(x) > f(x_0)$ for all $x \in (x_0, x_0 + \delta)$.

Question Type : **MSQ**
Question ID : **111686335**
Status : **Answered**
Chosen Option : **1,2,4**

Q.6 Let $m > 1$ and $n > 1$ be integers. Let A be an $m \times n$ matrix such that for some $m \times 1$ matrix b_1, the equation $Ax = b_1$ has infinitely many solutions. Let b_2 denote an $m \times 1$ matrix different from b_1. Then $Ax = b_2$ has

Options

1. infinitely many solutions for some b_2.
2. finitely many solutions for some b_2.
3. a unique solution for some b_2.
4. no solution for some b_2.

Question Type : **MSQ**
Question ID : **111686340**
Status : **Answered**
Chosen Option : **1,4**

Q.7 Which of the following subsets of $\mathbb{R}$ is/are connected?

Options

1. The set $\{x \in \mathbb{R} : x^3 - 2x + 1 \geq 0\}$.
2. The set $\{x \in \mathbb{R} : x^3 - 1 \geq 0\}$.
3. The set $\{x \in \mathbb{R} : x^3 + x + 1 \geq 0\}$.
4. The set $\{x \in \mathbb{R} : x \text{ is irrational}\}$.

Question Type : **MSQ**
Question ID : **111686337**
Status : **Answered**
Chosen Option : **1,2**

Q.8 Let $f : \mathbb{R} \to \mathbb{R}$ be a function with the property that for every $y \in \mathbb{R}$, the value of the expression

$$\sup_{x \in \mathbb{R}} [xy - f(x)]$$

is finite. Define $g(y) = \sup_{x \in \mathbb{R}} [xy - f(x)]$ for $y \in \mathbb{R}$. Then

Options

1. f must satisfy $\lim\limits_{|x| \to \infty} \dfrac{f(x)}{|x|} = +\infty.$
2. g is even if f is even.
3. g is odd if f is even.
4. f must satisfy $\lim\limits_{|x| \to \infty} \dfrac{f(x)}{|x|} = -\infty.$

Question Type : **MSQ**
Question ID : **111686331**
Status : **Answered**
Chosen Option : **1,2**

Q.9 Let G be a finite group of order 28. Assume that G contains a subgroup of order 7. Which of the following statements is/are true?

Options

1. G contains at least two subgroups of order 7.
2. G contains a normal subgroup of order 7.
3. G contains no normal subgroup of order 7.
4. G contains a unique subgroup of order 7.

Question Type : **MSQ**
Question ID : **111686336**
Status : **Answered**
Chosen Option : **2,4**

Q.10 Let $D = \mathbb{R}^2 \setminus \{(0,0)\}$. Consider the two functions $u, v : D \to \mathbb{R}$ defined by

$$u(x,y) = x^2 - y^2 \text{ and } v(x,y) = xy.$$

Consider the gradients ∇u and ∇v of the functions u and v, respectively. Then

Options

1. ∇u and ∇v are parallel at each point (x, y) of D.
2. ∇u and ∇v do not exist at some points (x, y) of D.
3. ∇u and ∇v at each point (x, y) of D span $\mathbb{R}^2$.
4. ∇u and ∇v are perpendicular at each point (x, y) of D.

Question Type : **MSQ**
Question ID : **111686333**
Status : **Answered**
Chosen Option : **3,4**

Section : **Section C**

Q.1 Let V be the real vector space of all continuous functions $f : [0,2] \to \mathbb{R}$ such that the restriction of f to the interval $[0,1]$ is a polynomial of degree less than or equal to 2, the restriction of f to the interval $[1,2]$ is a polynomial of degree less than or equal to 3 and $f(0) = 0$. Then the dimension of V is equal to _____.

Given Answer : **1**

Question Type : **NAT**
Question ID : **111686346**
Status : **Answered**

Q.2 Let $y : \left(\frac{9}{10}, 3\right) \to \mathbb{R}$ be a differentiable function satisfying

$$(x - 2y)\frac{dy}{dx} + (2x + y) = 0, \quad x \in \left(\frac{9}{10}, 3\right), \quad \text{and } y(1) = 1.$$

Then $y(2)$ equals _____.

Given Answer : **3**

Question Type : **NAT**
Question ID : **111686348**
Status : **Answered**

Q.3 The number of cycles of length 4 in S_6 is ______.

Given Answer : **90**

Question Type : **NAT**
Question ID : **111686341**
Status : **Answered**

Q.4 The value of

$$\frac{\pi}{2}\lim_{n\to\infty}\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right)\cdots\cos\left(\frac{\pi}{2^{n+1}}\right)$$

is ______.

Given Answer : **0.5**

Question Type : **NAT**
Question ID : **111686350**
Status : **Answered**

Q.5 Let $\vec{F} = (y+1)e^y\cos(x)\hat{i} + (y+2)e^y\sin(x)\hat{j}$ be a vector field in $\mathbb{R}^2$ and C be a continuously differentiable path with the starting point $(0,1)$ and the end point $(\frac{\pi}{2}, 0)$. Then

$$\int_C \vec{F}\cdot d\vec{r}$$

equals _____.

Given Answer : **1**

Question Type : **NAT**
Question ID : **111686349**
Status : **Answered**

Q.6 Consider the subset $S = \{(x,y) : x^2+y^2 > 0\}$ of $\mathbb{R}^2$. Let

$$P(x,y) = \frac{y}{x^2+y^2} \text{ and } Q(x,y) = -\frac{x}{x^2+y^2}$$

for $(x,y)\in S$. If C denotes the unit circle traversed in the counter-clockwise direction, then the value of

$$\frac{1}{\pi}\int_C (Pdx + Qdy)$$

is _____.

Given Answer : **-2**

Question Type : **NAT**
Question ID : **111686344**
Status : **Answered**

Q.7 The value of

$$\lim_{n\to\infty} \left(3^n + 5^n + 7^n\right)^{\frac{1}{n}}$$

is ______.

Given Answer : **7**

Question Type : **NAT**
Question ID : **111686342**
Status : **Answered**

Q.8 Consider the set $A = \{a \in \mathbb{R} : x^2 = a(a+1)(a+2)$ has a real root $\}$. The number of connected components of A is _____.

Given Answer : **1**

Question Type : **NAT**
Question ID : **111686345**
Status : **Answered**

Q.9 The number of group homomorphisms from the group $\mathbb{Z}_4$ to the group S_3 is _____.

Given Answer : **6**

Question Type : **NAT**
Question ID : **111686347**
Status : **Answered**

Q.10 Let $B = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$ and define $u(x, y, z) = \sin\left((1 - x^2 - y^2 - z^2)^2\right)$ for $(x, y, z) \in B$. Then the value of

$$\iiint_B \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) dx dy dz$$

is _____.

Given Answer : **0**

Question Type : **NAT**
Question ID : **111686343**
Status : **Answered**

Q.11 The least possible value of k, accurate up to two decimal places, for which the following problem

$$y''(t) + 2y'(t) + ky(t) = 0, t \in \mathbb{R},$$
$$y(0) = 0, y(1) = 0, y(1/2) = 1,$$

has a solution is _____.

Given Answer : **0.56**

Question Type : **NAT**
Question ID : **111686352**
Status : **Answered**

Q.12 Let S be the surface defined by

$$\{(x, y, z) \in \mathbb{R}^3 : z = 1 - x^2 - y^2,\ z \geq 0\}.$$

Let $\vec{F} = -y\hat{i} + (x-1)\hat{j} + z^2\hat{k}$ and $\hat{n}$ be the continuous unit normal field to the surface S with positive z-component. Then the value of

$$\frac{1}{\pi}\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$$

is _____.

Given Answer : **0**

Question Type : **NAT**
Question ID : **111686358**
Status : **Answered**

Q.13 Define the sequence

$$s_n = \begin{cases} \dfrac{1}{2^n}\displaystyle\sum_{j=0}^{n-2} 2^{2j} & \text{if } n > 0 \text{ is even,} \\ \\ \dfrac{1}{2^n}\displaystyle\sum_{j=0}^{n-1} 2^{2j} & \text{if } n > 0 \text{ is odd.} \end{cases}$$

Define $\sigma_m = \dfrac{1}{m}\displaystyle\sum_{n=1}^{m} s_n$. The number of limit points of the sequence $\{\sigma_m\}$ is _____.

Given Answer : **1**

Question Type : **NAT**
Question ID : **111686355**
Status : **Answered**

Q.14 The value of

$$\lim_{n\to\infty}\int_0^1 e^{x^2}\sin(nx)\,dx$$

is ______.

Given Answer : **2.71**

Question Type : **NAT**
Question ID : **111686357**
Status : **Answered**

Q.15 Consider those continuous functions $f:\mathbb{R}\to\mathbb{R}$ that have the property that given any $x\in\mathbb{R}$,

$$f(x)\in\mathbb{Q} \text{ if and only if } f(x+1)\in\mathbb{R}\setminus\mathbb{Q}.$$

The number of such functions is _____.

Given Answer : **0**

Question Type : **NAT**
Question ID : **111686353**
Status : **Answered**

Q.16 The number of elements of order two in the group S_4 is equal to _____.

Given Answer : **9**

Question Type : **NAT**
Question ID : **111686351**
Status : **Answered**

Q.17 The determinant of the matrix

$$\begin{pmatrix} 2021 & 2020 & 2020 & 2020 \\ 2021 & 2021 & 2020 & 2020 \\ 2021 & 2021 & 2021 & 2020 \\ 2021 & 2021 & 2021 & 2021 \end{pmatrix}$$

is ______.

Given Answer : **2021**

Question Type : **NAT**
Question ID : **111686356**
Status : **Answered**

Q.18

Let $A = \begin{pmatrix} 2 & -1 & 3 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{pmatrix}$. Then the largest eigenvalue of A is _____.

Given **4**
Answer :

Question Type : **NAT**
Question ID : **111686359**
Status : **Answered**

Q.19

The largest positive number a such that

$$\int_0^5 f(x)dx + \int_0^3 f^{-1}(x)dx \geq a$$

for every strictly increasing surjective continuous function $f : [0, \infty) \to [0, \infty)$ is _____.

Given **17**
Answer :

Question Type : **NAT**
Question ID : **111686354**
Status : **Answered**

Q.20

Let $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$. Consider the linear map T_A from the real vector space $M_4(\mathbb{R})$ to itself defined by $T_A(X) = AX - XA$, for all $X \in M_4(\mathbb{R})$. The dimension of the range of T_A is _____.

Given **8**
Answer :

Question Type : **NAT**
Question ID : **111686360**
Status : **Answered**