

KEY

Section 1: Algebra

1.1 a,b,c

1.2 a,c

1.3 b

1.4 $(p - 1)/2$

1.5

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

1.6

$$\begin{bmatrix} 1 & -2 & 2 & 0 \\ 0 & 1 & -4 & 6 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.7 $x^3 + 6x^2 + 18x + 24$

1.8 (a) n; (b) 0

1.9 a

1.10 b,c

Section 2: Analysis

2.1 b,c

2.2 $2f(0)$

2.3 $3[f(1/3) - f(0)]$

2.4 (a) 0; (b) $\frac{1}{2} \int_{-\pi}^{\pi} f(t) dt$

2.5 (a) Not uniformly convergent; (b) uniformly convergent; (c) not uniformly convergent

2.6

$$\frac{2}{x} \left(e^{x^2} - e^{-x^2} \right)$$

2.7

$$e^{\frac{k(k+1)}{2a}}$$

2.8

$$\frac{1}{4} + \frac{1}{4} \sum_{n=1}^{\infty} (-1)^n (n+1) \left(\frac{z-2}{2} \right)^n$$

2.9 0

2.10 b

Section 3: Topology

3.1 b,c

3.2 a,b

3.3 b,c

3.4 a,b

3.5 b

3.6 b

3.7 none

3.8 a,c

3.9 c

3.10 a,c

Section 4: Applied Mathematics

4.1 $x^2 + 2y^2 = c^2$

4.2 $-u'' = f$ on $]0, 1[$; $u(0) = u(1) = 0$

4.3

$x(t) = e^t(\cos \omega t - \sin \omega t)$
$y(t) = e^t(\cos \omega t + \sin \omega t)$

4.4 $u(x, t) = (x - bt)^2$

4.5 $v''(r) + \frac{n-1}{r} v'(r) = 0$

4.6

$$L[y](s) = \frac{1+a+s}{s^2+as+b}$$

4.7 $x_{n+1} = \frac{1}{2}(3x_n - ax_n^3)$

4.8 b,c

4.9 b

4.10 max $z = 21$ at $x = 3; y = 0$

Section 5: Miscellaneous

5.1 b,c

5.2 $\frac{N}{2}(2a + (N^2 - 1)d)$

5.3 $n! - 2(n-1)!$

5.4

$$D_n = n! - 1 - \sum_{k=1}^{n-2} \binom{n}{k} D_{n-k}$$

5.5 $44/120 = 11/30$

5.6 a,b,c

5.7 a,b,c

5.8

$$\frac{x}{1-x-x^2}$$

5.9 semi-major axis = 1; semi-minor axis = $1/3$

5.10

$$a + (b-a)e^{-\frac{1}{2}}$$

Note:

Accept any correct equivalent form of the answers.