

**Paper Specific Instructions**

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.
2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.
4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.
5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ)**, wrong answer will result in **NEGATIVE** marks. For all 1 mark questions,  $\frac{1}{3}$  marks will be deducted for each wrong answer. For all 2 marks questions,  $\frac{2}{3}$  marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.
6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.
7. The Scribble Pad will be provided for rough work.

## Notation

$\mathbb{N}$	set of all natural numbers $1, 2, 3, \dots$
$\mathbb{R}$	set of all real numbers
$M_{m \times n}(\mathbb{R})$	real vector space of all matrices of size $m \times n$ with entries in $\mathbb{R}$
$\emptyset$	empty set
$X \setminus Y$	set of all elements from the set $X$ which are not in the set $Y$
$\mathbb{Z}_n$	group of all congruence classes of integers modulo $n$
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive $x, y$ and $z$ axes of a three dimensional rectangular coordinate system, respectively
$S_n$	group of all permutations of the set $\{1, 2, 3, \dots, n\}$
$\ln$	logarithm to the base $e$
$\log$	logarithm to the base 10
$\nabla$	$\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$
$\det(M)$	determinant of a square matrix $M$

**SECTION – A**  
**MULTIPLE CHOICE QUESTIONS (MCQ)**

**Q. 1 – Q.10 carry one mark each.**

Q.1 Let  $a_1 = b_1 = 0$ , and for each  $n \geq 2$ , let  $a_n$  and  $b_n$  be real numbers given by

$$a_n = \sum_{m=2}^n \frac{(-1)^m m}{(\log(m))^m} \quad \text{and} \quad b_n = \sum_{m=2}^n \frac{1}{(\log(m))^m}.$$

Then which one of the following is TRUE about the sequences  $\{a_n\}$  and  $\{b_n\}$ ?

- (A) Both  $\{a_n\}$  and  $\{b_n\}$  are divergent
- (B)  $\{a_n\}$  is convergent and  $\{b_n\}$  is divergent
- (C)  $\{a_n\}$  is divergent and  $\{b_n\}$  is convergent
- (D) Both  $\{a_n\}$  and  $\{b_n\}$  are convergent

Q.2 Let  $T \in M_{m \times n}(\mathbb{R})$ . Let  $V$  be the subspace of  $M_{n \times p}(\mathbb{R})$  defined by

$$V = \{X \in M_{n \times p}(\mathbb{R}) : TX = 0\}.$$

Then the dimension of  $V$  is

- (A)  $pn - \text{rank}(T)$
- (B)  $mn - p \text{rank}(T)$
- (C)  $p(m - \text{rank}(T))$
- (D)  $p(n - \text{rank}(T))$

Q.3 Let  $g: \mathbb{R} \rightarrow \mathbb{R}$  be a twice differentiable function. Define  $f: \mathbb{R}^3 \rightarrow \mathbb{R}$  by

$$f(x, y, z) = g(x^2 + y^2 - 2z^2).$$

Then  $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$  is equal to

- (A)  $4(x^2 + y^2 - 4z^2) g''(x^2 + y^2 - 2z^2)$
- (B)  $4(x^2 + y^2 + 4z^2) g''(x^2 + y^2 - 2z^2)$
- (C)  $4(x^2 + y^2 - 2z^2) g''(x^2 + y^2 - 2z^2)$
- (D)  $4(x^2 + y^2 + 4z^2) g''(x^2 + y^2 - 2z^2) + 8g'(x^2 + y^2 - 2z^2)$

Q.4 Let  $\{a_n\}_{n=0}^{\infty}$  and  $\{b_n\}_{n=0}^{\infty}$  be sequences of positive real numbers such that  $na_n < b_n < n^2 a_n$  for all  $n \geq 2$ . If the radius of convergence of the power series  $\sum_{n=0}^{\infty} a_n x^n$  is 4, then the power series  $\sum_{n=0}^{\infty} b_n x^n$

- (A) converges for all  $x$  with  $|x| < 2$
- (B) converges for all  $x$  with  $|x| > 2$
- (C) does not converge for any  $x$  with  $|x| > 2$
- (D) does not converge for any  $x$  with  $|x| < 2$

Q.5 Let  $S$  be the set of all limit points of the set  $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$ . Let  $\mathbb{Q}_+$  be the set of all positive rational numbers. Then

- (A)  $\mathbb{Q}_+ \subseteq S$
- (B)  $S \subseteq \mathbb{Q}_+$
- (C)  $S \cap (\mathbb{R} \setminus \mathbb{Q}_+) \neq \emptyset$
- (D)  $S \cap \mathbb{Q}_+ \neq \emptyset$

Q.6 If  $x^h y^k$  is an integrating factor of the differential equation

$$y(1 + xy) dx + x(1 - xy) dy = 0,$$

then the ordered pair  $(h, k)$  is equal to

- (A)  $(-2, -2)$       (B)  $(-2, -1)$       (C)  $(-1, -2)$       (D)  $(-1, -1)$

Q.7 If  $y(x) = \lambda e^{2x} + e^{\beta x}$ ,  $\beta \neq 2$ , is a solution of the differential equation

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

satisfying  $\frac{dy}{dx}(0) = 5$ , then  $y(0)$  is equal to

- (A) 1      (B) 4      (C) 5      (D) 9

Q.8 The equation of the tangent plane to the surface  $x^2 z + \sqrt{8 - x^2 - y^4} = 6$  at the point  $(2, 0, 1)$  is

- (A)  $2x + z = 5$       (B)  $3x + 4z = 10$   
 (C)  $3x - z = 10$       (D)  $7x - 4z = 10$

Q.9 The value of the integral

$$\int_{y=0}^1 \int_{x=0}^{1-y^2} y \sin(\pi(1-x)^2) dx dy$$

is

- (A)  $\frac{1}{2\pi}$       (B)  $2\pi$       (C)  $\frac{\pi}{2}$       (D)  $\frac{2}{\pi}$

Q.10 The area of the surface generated by rotating the curve  $x = y^3$ ,  $0 \leq y \leq 1$ , about the  $y$ -axis, is

- (A)  $\frac{\pi}{27} 10^{3/2}$       (B)  $\frac{4\pi}{3} (10^{3/2} - 1)$       (C)  $\frac{\pi}{27} (10^{3/2} - 1)$       (D)  $\frac{4\pi}{3} 10^{3/2}$

**Q. 11 – Q. 30 carry two marks each.**

Q.11 Let  $H$  and  $K$  be subgroups of  $\mathbb{Z}_{144}$ . If the order of  $H$  is 24 and the order of  $K$  is 36, then the order of the subgroup  $H \cap K$  is

- (A) 3      (B) 4      (C) 6      (D) 12

Q.12 Let  $P$  be a  $4 \times 4$  matrix with entries from the set of rational numbers. If  $\sqrt{2} + i$ , with  $i = \sqrt{-1}$ , is a root of the characteristic polynomial of  $P$  and  $I$  is the  $4 \times 4$  identity matrix, then

- (A)  $P^4 = 4P^2 + 9I$       (B)  $P^4 = 4P^2 - 9I$       (C)  $P^4 = 2P^2 - 9I$       (D)  $P^4 = 2P^2 + 9I$

- Q.13 The set  $\left\{\frac{x}{1+x} : -1 < x < 1\right\}$ , as a subset of  $\mathbb{R}$ , is
- (A) connected and compact  
 (B) connected but not compact  
 (C) not connected but compact  
 (D) neither connected nor compact
- Q.14 The set  $\left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\} \cup \{0\}$ , as a subset of  $\mathbb{R}$ , is
- (A) compact and open  
 (B) compact but not open  
 (C) not compact but open  
 (D) neither compact nor open
- Q.15 For  $-1 < x < 1$ , the sum of the power series  $1 + \sum_{n=2}^{\infty} (-1)^{n-1} n^2 x^{n-1}$  is
- (A)  $\frac{1-x}{(1+x)^3}$   
 (B)  $\frac{1+x^2}{(1+x)^4}$   
 (C)  $\frac{1-x}{(1+x)^2}$   
 (D)  $\frac{1+x^2}{(1+x)^3}$
- Q.16 Let  $f(x) = (\ln x)^2$ ,  $x > 0$ . Then
- (A)  $\lim_{x \rightarrow \infty} \frac{f(x)}{x}$  does not exist  
 (B)  $\lim_{x \rightarrow \infty} f'(x) = 2$   
 (C)  $\lim_{x \rightarrow \infty} (f(x+1) - f(x)) = 0$   
 (D)  $\lim_{x \rightarrow \infty} (f(x+1) - f(x))$  does not exist
- Q.17 Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(x) > f(x)$  for all  $x \in \mathbb{R}$ , and  $f(0) = 1$ . Then  $f(1)$  lies in the interval
- (A)  $(0, e^{-1})$       (B)  $(e^{-1}, \sqrt{e})$       (C)  $(\sqrt{e}, e)$       (D)  $(e, \infty)$
- Q.18 For which one of the following values of  $k$ , the equation
- $$2x^3 + 3x^2 - 12x - k = 0$$
- has three distinct real roots?
- (A) 16      (B) 20      (C) 26      (D) 31
- Q.19 Which one of the following series is divergent?
- (A)  $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$   
 (B)  $\sum_{n=1}^{\infty} \frac{1}{n} \log n$   
 (C)  $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$   
 (D)  $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$

Q.20 Let  $S$  be the family of orthogonal trajectories of the family of curves

$$2x^2 + y^2 = k, \text{ for } k \in \mathbb{R} \text{ and } k > 0.$$

If  $C \in S$  and  $C$  passes through the point  $(1, 2)$ , then  $C$  also passes through

- (A)  $(4, -\sqrt{2})$       (B)  $(2, -4)$       (C)  $(2, 2\sqrt{2})$       (D)  $(4, 2\sqrt{2})$

Q.21 Let  $x$ ,  $x + e^x$  and  $1 + x + e^x$  be solutions of a linear second order ordinary differential equation with constant coefficients. If  $y(x)$  is the solution of the same equation satisfying  $y(0) = 3$  and  $y'(0) = 4$ , then  $y(1)$  is equal to

- (A)  $e + 1$       (B)  $2e + 3$       (C)  $3e + 2$       (D)  $3e + 1$

Q.22 The function

$$f(x, y) = x^3 + 2xy + y^3$$

has a saddle point at

- (A)  $(0, 0)$       (B)  $\left(-\frac{2}{3}, -\frac{2}{3}\right)$       (C)  $\left(-\frac{3}{2}, -\frac{3}{2}\right)$       (D)  $(-1, -1)$

Q.23 The area of the part of the surface of the paraboloid  $x^2 + y^2 + z = 8$  lying inside the cylinder  $x^2 + y^2 = 4$  is

- (A)  $\frac{\pi}{2}(17^{3/2} - 1)$       (B)  $\pi(17^{3/2} - 1)$       (C)  $\frac{\pi}{6}(17^{3/2} - 1)$       (D)  $\frac{\pi}{3}(17^{3/2} - 1)$

Q.24 Let  $C$  be the circle  $(x - 1)^2 + y^2 = 1$ , oriented counter clockwise. Then the value of the line integral

$$\oint_C -\frac{4}{3}xy^3 dx + x^4 dy$$

is

- (A)  $6\pi$       (B)  $8\pi$       (C)  $12\pi$       (D)  $14\pi$

Q.25 Let  $\vec{F}(x, y, z) = 2y \hat{i} + x^2 \hat{j} + xy \hat{k}$  and let  $C$  be the curve of intersection of the plane  $x + y + z = 1$  and the cylinder  $x^2 + y^2 = 1$ . Then the value of

$$\left| \oint_C \vec{F} \cdot d\vec{r} \right|$$

is

- (A)  $\pi$       (B)  $\frac{3\pi}{2}$       (C)  $2\pi$       (D)  $3\pi$

- Q.26 The tangent line to the curve of intersection of the surface  $x^2 + y^2 - z = 0$  and the plane  $x + z = 3$  at the point  $(1, 1, 2)$  passes through
- (A)  $(-1, -2, 4)$       (B)  $(-1, 4, 4)$       (C)  $(3, 4, 4)$       (D)  $(-1, 4, 0)$

- Q.27 The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of  $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ ?
- (A)  $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$       (B)  $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$       (C)  $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$       (D)  $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

- Q.28 Let  $\{a_n\}$  be a sequence of positive real numbers. The series  $\sum_{n=1}^{\infty} a_n$  converges if the series
- (A)  $\sum_{n=1}^{\infty} a_n^2$  converges  
 (B)  $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$  converges  
 (C)  $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$  converges  
 (D)  $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$  converges

- Q.29 For  $\beta \in \mathbb{R}$ , define

$$f(x, y) = \begin{cases} \frac{x^2|x|^\beta y}{x^4 + y^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then, at  $(0, 0)$ , the function  $f$  is

- (A) continuous for  $\beta = 0$   
 (B) continuous for  $\beta > 0$   
 (C) not differentiable for any  $\beta$   
 (D) continuous for  $\beta < 0$
- Q.30 Let  $\{a_n\}$  be a sequence of positive real numbers such that

$$a_1 = 1, \quad a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0 \text{ for all } n \geq 1.$$

Then the sum of the series  $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$  lies in the interval

- (A)  $(1, 2]$       (B)  $(2, 3]$       (C)  $(3, 4]$       (D)  $(4, 5]$

## SECTION - B

## MULTIPLE SELECT QUESTIONS (MSQ)

**Q. 31 – Q. 40 carry two marks each.**

Q.31 Let  $G$  be a noncyclic group of order 4. Consider the statements I and II:

- I. There is NO injective (one-one) homomorphism from  $G$  to  $\mathbb{Z}_8$   
 II. There is NO surjective (onto) homomorphism from  $\mathbb{Z}_8$  to  $G$

Then

- (A) I is true (B) I is false  
 (C) II is true (D) II is false

Q.32 Let  $G$  be a nonabelian group,  $y \in G$ , and let the maps  $f, g, h$  from  $G$  to itself be defined by

$$f(x) = yxy^{-1}, \quad g(x) = x^{-1} \quad \text{and} \quad h = g \circ g.$$

Then

- (A)  $g$  and  $h$  are homomorphisms and  $f$  is not a homomorphism  
 (B)  $h$  is a homomorphism and  $g$  is not a homomorphism  
 (C)  $f$  is a homomorphism and  $g$  is not a homomorphism  
 (D)  $f, g$  and  $h$  are homomorphisms

Q.33 Let  $S$  and  $T$  be linear transformations from a finite dimensional vector space  $V$  to itself such that  $S(T(v)) = 0$  for all  $v \in V$ . Then

- (A)  $\text{rank}(T) \geq \text{nullity}(S)$  (B)  $\text{rank}(S) \geq \text{nullity}(T)$   
 (C)  $\text{rank}(T) \leq \text{nullity}(S)$  (D)  $\text{rank}(S) \leq \text{nullity}(T)$

Q.34 Let  $\vec{F}$  and  $\vec{G}$  be differentiable vector fields and let  $g$  be a differentiable scalar function. Then

- (A)  $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} - \vec{F} \cdot \nabla \times \vec{G}$  (B)  $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} + \vec{F} \cdot \nabla \times \vec{G}$   
 (C)  $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} - \nabla g \cdot \vec{F}$  (D)  $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} + \nabla g \cdot \vec{F}$

Q.35 Consider the intervals  $S = (0, 2]$  and  $T = [1, 3)$ . Let  $S^\circ$  and  $T^\circ$  be the sets of interior points of  $S$  and  $T$ , respectively. Then the set of interior points of  $S \setminus T$  is equal to

- (A)  $S \setminus T^\circ$  (B)  $S \setminus T$  (C)  $S^\circ \setminus T^\circ$  (D)  $S^\circ \setminus T$

Q.36 Let  $\{a_n\}$  be the sequence given by

$$a_n = \max \left\{ \sin \left( \frac{n\pi}{3} \right), \cos \left( \frac{n\pi}{3} \right) \right\}, \quad n \geq 1.$$

Then which of the following statements is/are TRUE about the subsequences  $\{a_{6n-1}\}$  and  $\{a_{6n+4}\}$ ?

- (A) Both the subsequences are convergent  
 (B) Only one of the subsequences is convergent  
 (C)  $\{a_{6n-1}\}$  converges to  $-\frac{1}{2}$   
 (D)  $\{a_{6n+4}\}$  converges to  $\frac{1}{2}$



Q.37 Let

$$f(x) = \cos(|\pi - x|) + (x - \pi) \sin |x| \text{ and } g(x) = x^2 \text{ for } x \in \mathbb{R}.$$

If  $h(x) = f(g(x))$ , then

- (A)  $h$  is not differentiable at  $x = 0$
- (B)  $h'(\sqrt{\pi}) = 0$
- (C)  $h''(x) = 0$  has a solution in  $(-\pi, \pi)$
- (D) there exists  $x_0 \in (-\pi, \pi)$  such that  $h(x_0) = x_0$

Q.38 Let  $f: (0, \frac{\pi}{2}) \rightarrow \mathbb{R}$  be given by

$$f(x) = (\sin x)^\pi - \pi \sin x + \pi.$$

Then which of the following statements is/are TRUE?

- (A)  $f$  is an increasing function
- (B)  $f$  is a decreasing function
- (C)  $f(x) > 0$  for all  $x \in (0, \frac{\pi}{2})$
- (D)  $f(x) < 0$  for some  $x \in (0, \frac{\pi}{2})$

Q.39 Let

$$f(x, y) = \begin{cases} \frac{|x|}{|x| + |y|} \sqrt{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$$

Then at  $(0, 0)$ ,

- (A)  $f$  is continuous
- (B)  $\frac{\partial f}{\partial x} = 0$  and  $\frac{\partial f}{\partial y}$  does not exist
- (C)  $\frac{\partial f}{\partial x}$  does not exist and  $\frac{\partial f}{\partial y} = 0$
- (D)  $\frac{\partial f}{\partial x} = 0$  and  $\frac{\partial f}{\partial y} = 0$

Q.40 Let  $\{a_n\}$  be the sequence of real numbers such that

$$a_1 = 1 \text{ and } a_{n+1} = a_n + a_n^2 \text{ for all } n \geq 1.$$

Then

- (A)  $a_4 = a_1(1 + a_1)(1 + a_2)(1 + a_3)$
- (B)  $\lim_{n \rightarrow \infty} \frac{1}{a_n} = 0$
- (C)  $\lim_{n \rightarrow \infty} \frac{1}{a_n} = 1$
- (D)  $\lim_{n \rightarrow \infty} a_n = 0$

## SECTION – C

## NUMERICAL ANSWER TYPE (NAT)

**Q. 41 – Q. 50 carry one mark each.**

Q.41 Let  $x$  be the 100-cycle  $(1\ 2\ 3\ \dots\ 100)$  and let  $y$  be the transposition  $(49\ 50)$  in the permutation group  $S_{100}$ . Then the order of  $xy$  is \_\_\_\_\_

Q.42 Let  $W_1$  and  $W_2$  be subspaces of the real vector space  $\mathbb{R}^{100}$  defined by

$$W_1 = \{ (x_1, x_2, \dots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 4 \},$$

$$W_2 = \{ (x_1, x_2, \dots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 5 \}.$$

Then the dimension of  $W_1 \cap W_2$  is \_\_\_\_\_

Q.43 Consider the following system of three linear equations in four unknowns  $x_1, x_2, x_3$  and  $x_4$

$$x_1 + x_2 + x_3 + x_4 = 4,$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 5,$$

$$x_1 + 3x_2 + 5x_3 + kx_4 = 5.$$

If the system has no solutions, then  $k =$  \_\_\_\_\_

Q.44 Let  $\vec{F}(x, y) = -y\hat{i} + x\hat{j}$  and let  $C$  be the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

oriented counter clockwise. Then the value of  $\oint_C \vec{F} \cdot d\vec{r}$  (round off to 2 decimal places) is \_\_\_\_\_

Q.45 The coefficient of  $\left(x - \frac{\pi}{2}\right)$  in the Taylor series expansion of the function

$$f(x) = \begin{cases} \frac{4(1 - \sin x)}{2x - \pi}, & x \neq \frac{\pi}{2} \\ 0, & x = \frac{\pi}{2} \end{cases}$$

about  $x = \frac{\pi}{2}$ , is \_\_\_\_\_

Q.46 Let  $f: [0, 1] \rightarrow \mathbb{R}$  be given by

$$f(x) = \frac{\left(1+x^{\frac{1}{3}}\right)^3 + \left(1-x^{\frac{1}{3}}\right)^3}{8(1+x)}.$$

Then

$$\max \{f(x): x \in [0,1]\} - \min \{f(x): x \in [0,1]\}$$

is \_\_\_\_\_

Q.47 If

$$g(x) = \int_{x(x-2)}^{4x-5} f(t) dt, \text{ where } f(x) = \sqrt{1+3x^4} \text{ for } x \in \mathbb{R}$$

then  $g'(1) =$  \_\_\_\_\_

Q.48 Let

$$f(x, y) = \begin{cases} \frac{x^3 + y^3}{x^2 - y^2}, & x^2 - y^2 \neq 0 \\ 0, & x^2 - y^2 = 0. \end{cases}$$

Then the directional derivative of  $f$  at  $(0, 0)$  in the direction of  $\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$  is \_\_\_\_\_

Q.49 The value of the integral

$$\int_{-1}^1 \int_{-1}^1 |x + y| dx dy$$

(round off to 2 decimal places) is \_\_\_\_\_

Q.50 The volume of the solid bounded by the surfaces  $x = 1 - y^2$  and  $x = y^2 - 1$ , and the planes  $z = 0$  and  $z = 2$  (round off to 2 decimal places) is \_\_\_\_\_

**Q. 51 – Q. 60 carry two marks each.**

Q.51 The volume of the solid of revolution of the loop of the curve  $y^2 = x^4(x + 2)$  about the  $x$ -axis (round off to 2 decimal places) is \_\_\_\_\_

Q.52 The greatest lower bound of the set

$$\{(e^n + 2^n)^{\frac{1}{n}} : n \in \mathbb{N}\},$$

(round off to 2 decimal places) is \_\_\_\_\_

Q.53 Let  $G = \{n \in \mathbb{N} : n \leq 55, \gcd(n, 55) = 1\}$  be the group under multiplication modulo 55. Let  $x \in G$  be such that  $x^2 = 26$  and  $x > 30$ . Then  $x$  is equal to \_\_\_\_\_

Q.54 The number of critical points of the function

$$f(x, y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$$

is \_\_\_\_\_

Q.55 The number of elements in the set  $\{x \in S_3 : x^4 = e\}$ , where  $e$  is the identity element of the permutation group  $S_3$ , is \_\_\_\_\_

Q.56 If  $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}$ ,  $y, z \in \mathbb{R}$ , is an eigenvector corresponding to a real eigenvalue of the matrix  $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$  then  $z - y$  is equal to \_\_\_\_\_

Q.57 Let  $M$  and  $N$  be any two  $4 \times 4$  matrices with integer entries satisfying

$$MN = 2 \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Then the maximum value of  $\det(M) + \det(N)$  is \_\_\_\_\_

Q.58 Let  $M$  be a  $3 \times 3$  matrix with real entries such that  $M^2 = M + 2I$ , where  $I$  denotes the  $3 \times 3$  identity matrix. If  $\alpha, \beta$  and  $\gamma$  are eigenvalues of  $M$  such that  $\alpha\beta\gamma = -4$ , then  $\alpha + \beta + \gamma$  is equal to \_\_\_\_\_

Q.59 Let  $y(x) = xv(x)$  be a solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 3y = 0.$$

If  $v(0) = 0$  and  $v(1) = 1$ , then  $v(-2)$  is equal to \_\_\_\_\_

Q.60 If  $y(x)$  is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, \quad y(0) = 2, \quad \frac{dy}{dx}(0) = 0,$$

then  $y(\ln 2)$  is (round off to 2 decimal places) equal to \_\_\_\_\_

**END OF THE QUESTION PAPER**