CALCULUS OF VARIATION ASSIGNMENT

JUNE - 2014

PART – B

1. The curve eternizing the functional $I(y)=\int_1^2 \frac{\sqrt{1+(y'(x))^2}}{x}dx,$ y(1)=0,y(2)=1 is

1. an ellipse
2. a parabola
3. a circle
4. a straight line

PART – C

2. Let u(x,y) be an extremal of the functional $J(u)=\iint_D\left[\frac{1}{2}u_x^2+\frac{1}{2}u_y^2+e^{xy}u\right]dxdy,$ where D is the open unit disk in $\mathbb{R}^2$. Then u satisfies

1. $u_{xx}+u_{yy}-e^{x+y}=0$
2. $u_{xx}+u_{yy}=e^{xy}$
3. $u_{xx}+u_{yy}=-e^{xy}$
4. $\iint_D\left[u_{xx}+u_{yy}-e^{xy}\right]h(x,y)dxdy=0$ for every smooth h vanishing on the boundary of D

DECEMBER – 2014

PART - B

3. Consider the functional

$$I(y)=y^2(1)+\int_0^1 y'^2(x)dx,\ y(0)=1,$$

where $y\in C^2([0,1])$. If y extremizes J, then

1. $y(x)=1-\frac{1}{2}x^2$
2. $y(x)=1-\frac{1}{2}x$
3. $y(x)=1+\frac{1}{2}x$
4. $y(x)=1+\frac{1}{2}x^2$

PART - C

4. Let $y\in C^2([0,\pi])$ satisfying $y(0)=y(\pi)=0$ and $\int_0^\pi y^2(x)dx=1$ extremize the functional

$$J(y)=\int_0^\pi (y'(x))^2dx;\ y'=\frac{dy}{dx}.$$ Then

1. $y(x)=\sqrt{\frac{2}{\pi}}\sin x$
2. $y(x)=-\sqrt{\frac{2}{\pi}}\sin x$
3. $y(x)=\sqrt{\frac{2}{\pi}}\cos x$
4. $y(x)=-\sqrt{\frac{2}{\pi}}\cos x$

JUNE – 2015

PART - C

5. The extremal of the functional $\int_0^\alpha (y'^2-y^2)dx$ that passes through (0,0) and (α,0) has a

1. weak minimum if $\alpha < \pi$.
2. strong minimum if $\alpha < \pi$.
3. weak minimum if $\alpha > \pi$.
4. strong minimum if $\alpha > \pi$.

6. The extremal of the functional $I=\int_0^{x_1} y^2(y')^2dx$ that passes through (0,0) and (x_1, y_1) is

1. a constant function
2. a linear function of x
3. part of parabola
4. part of an ellipse

DECEMBER – 2015

PART - B

7. The functional $I(y(x))=\int_a^b(y^2+y'^2-2y\sin x)dx,$ has the following extremal with C_1 and C_2 as arbitrary constants.

1. $y=C_1e^{2x}+C_2e^{-2x}+\frac{1}{2}\sin x.$
2. $y=C_1e^{x}+C_2e^{-x}+\frac{1}{2}\sin x.$
3. $y=C_1e^{x}+C_2e^{-x}-\frac{1}{2}\sin x.$
4. $y=C_1e^{2x}+C_2e^{-2x}+\frac{1}{2}\cos x.$

PART - C

8. To show the existence of a minimizer for the functional $J[y]=\int_a^b f(x,y,y')\,dx,$ for which there is a minimizing sequence (φ_n), it is enough to have

1. (φ_n) is convergent and J is continuous.
2. (φ_n) is convergent and J is differentiable.
3. (φ_n) has a convergent subsequence and J is continuous.
4. (φ_n) has a convergent subsequence and J is differentiable.

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JUNE – 2016

PART – B

9. The curve of fixed length l, that joins the points (0,0) and (1,0), lies above the x-axis, and encloses the maximum area between itself and the x-axis, is a segment of
1. a straight line. 2. a parabola
3. an ellipse 4. a circle

PART - C

10. Let y = y(x) be the extremal of the functional $I[y(x)] = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx,$ subject to the condition that the left end of the extremal moves along y = x^2, while the right end moves along x – y = 5, Then the
1. shortest distance between the parabola and the straight line is $\left(\frac{19\sqrt{2}}{8}\right)$.
2. slope of the extremal at (x,y) is $\left(-\frac{3}{2}\right)$.
3. point $\left(\frac{3}{4},0\right)$ lies on the extremal.
4. extremal is orthogonal to the curve $y=\frac{x}{2}$.

DECEMBER – 2016

PART - B

11. If $J[y]=\int_1^2 (y'^2+2yy'+y^2)\,dx,$ y(1) = 1 and y(2) is arbitrary then the extremal is
1. e^{x-1} 2. e^{x+1}
3. e^{1-x} 4. e^{-x-1}

PART - C

12. The functional $J[y]=\int_0^1 (y'^2+x^2)dx$ where y(0)=-1 and y(1) = 1 on y=2 x -1, has
1. weak minimum
2. weak maximum
3. strong minimum
4. strong maximum

13. Let $y(x)$ be a piecewise continuously differentiable function on [0,4]. Then the functional $J[y]=\int_0^4 (y'-1)^2(y'+1)^2\,dx$ attains minimum if $y = y(x)$ is
1. $y=\frac{x}{2} \quad 0\le x\le 4$
2. $y=\begin{cases}-x & 0\le x\le 1\\ x-2 & 1\le x\le 4\end{cases}$
3. $y=\begin{cases}2x & 0\le x\le 2\\ -x+6 & 2\le x\le 4\end{cases}$
4. $y=\begin{cases}x & 0\le x\le 3\\ -x+6 & 3\le x\le 4\end{cases}$

JUNE – 2017

PART – B

14. The infimum of $\int_0^1 (u'(t))^2\,dt$ on the class of functions $\{u\in C^1[0,1]\text{such that } u(0)=0 \text{ and } \max_{[0,1]}|u|=1\}$ is equal to
1. 0 2. 1/2
3. 1 4. 2

15. Consider the functional $I(y(x))=\int_{x_0}^{x_1} f(x,y)\sqrt{1+y'^2}\,e^{\tan^{-1}y'}dx$ where f(x,y)≠0. Let the left end of the extremal be fixed at the point A(x_0, y_0) and the right end B(x_1,y_1) be movable along the curve y=ψ(x). Then the extremal y=y(x) intersects the curve y=ψ(x) along which the boundary point B(x_1,y_1) slides at an angle
1. π/3 2. π/2
3. π/4 4. π/6

DECEMBER – 2017

PART – C

16. Let X={u∈C^1[0,1] | u(0)=0} and let I:X→ℝ be defined as $I(u)=\int_0^1 (u'(t)^2-u(t)^2)dt$.
Which of the following are correct?
1. I is bounded below
2. I is not bounded below
3. I attains its infimum

4. I does not attain its infimum

17. Let I : C^1 [0,1] →ℝ be defined as $I(u)=\frac{1}{2}\int_0^1(u'(t)^2-4\pi^2u(t)^2)dt$.

Let us set (P)m = inf {I (u) : u ∈ C^1[0,1] : u(0) = u(1) = 0}. Let $\bar{u}\in C^1[0,1]$ satisfy the Euler – Lagrange Equation associated with (P). Then
1. m= -∞ i.e., I is not bounded below
2. m∈ℝ, with I($\bar{u}$) = m
3. m∈ℝ, with I($\bar{u}$) > m
4. m∈ℝ, with I($\bar{u}$) < m

JUNE - 2018

PART - B

18. Consider $J[y]=\int_0^1[(y')^2+2y]dx$ subject to $y(0)=0, y(1)=1$. Then inf J[y]

1. is $\frac{23}{12}$ 2. is $\frac{21}{24}$
3. is $\frac{18}{25}$ 4. does not exist

PART - C

19. The extremal of the functional $J[y]=\int_0^1 y'^2(x)\,dx$ subject to y(0)=0, y(1)= 1 and $\int_0^1 y(x)\,dx=0$ is
1. $3x^2 - 2x$
2. $8x^3 - 9x^2 + 2x$
3. $\frac{5}{3}x^4-\frac{2}{3}x$
4. $\frac{-21}{2}x^5+10x^4+4x^3-\frac{5}{2}x$

20. The admissible extremal for $J[y]=\int_0^{\log 3}[e^{-x}y'^2+2e^x(y'+y)]\,dx,$ where y (log 3) = 1 and y(0) is free is
1. $4 - e^x$ 2. $10 - e^{2x}$
3. $e^x - 2$ 4. $e^{2x} - 8$

DECEMBER – 2018

PART-B

21. Consider the functional $J[y]=\int_0^2(1-y'^2)^2dx$ defined on {y∈C[0,2]: y is piecewise C^1 and y(0)=y(2) = 0}. Let y_e be a minimize of the above functional. Then y_e has
1. a unique corner point
2. two corner points
3. more than two corner points
4. no corner points

PART-C

22. Consider the functional $J[y]=\int_0^1[(y')^2-(y')^4]dx$ subject to y(0)=0, y(1) = 0. A broken extremal is a continuous extremal whose derivative has jump discontinuities at a finite number of points. Then which of the following statements are true?
1. There are no broken extremals and y = 0 is an extremal.
2. There is a unique broken extremal.
3. There exist more than one and finitely many broken extremals.
4. There exist infinitely many broken extremals.

23. The extremals of the functional $J[y]=\int_0^1[720x^2y-(y'')^2]dx,$ subject to y(x) = y′(0) = y(1) = 0, y′(1) = 6, are
1. $x^6 + 2x^3 - 3x^2$ 2. $x^5 + 4x^4 - 5x^3$
3. $x^5 + x^4 - 2x^3$ 4. $x^6 + 4x^3 - 6x^2$

JUNE – 19

PART – B

24. Let x*(t) be the curve which minimizes the functional $J(x)=\int_0^1[x^2(t)+\dot{x}^2(t)]\,dt$ satisfying x(0)=0, x(1) = 1. Then the value of $x^*\left(\frac{1}{2}\right)$ is

1. $\frac{\sqrt{e}}{1+e}$ 2. $\frac{2\sqrt{e}}{1+e}$
3. $\frac{\sqrt{e}}{1+2e}$ 4. $\frac{2\sqrt{e}}{1+2e}$

PART – C

25. Let B = {(x, y) ∈ $\mathbb{R}^2$|$x^2 + y^2 \le 1$} and C = {(x, y)|$x^2 + y^2 = 1$} and let f and g be continuous functions. Let u be the minimizer of the functional

$J[v]=\iint_B (v_x^2+v_y^2-2fv)\,dx\,dy+\int_C (v^2-2gv)\,ds.$

Then u is a solution of

1. $-\Delta u=f,\ \frac{\partial u}{\partial n}+u=g$
2. $\Delta u=f,\ \frac{\partial u}{\partial n}-u=g$
3. $-\Delta u=f,\ \frac{\partial u}{\partial n}=g$
4. $\Delta u=f,\ \frac{\partial u}{\partial n}=g$

where $\frac{\partial u}{\partial n}$ denotes the directional derivative of u in the direction of the outward drawn normal at (x, y) $\in$ C

26. Consider the functional $J[y]=\int_0^1\left[(y'(x))^2+(y'(x))^3\right]dx,$ subject to y(0) = 1 and y(1) = 2. Then

1. there exists an extremal y $\in$ C^1 ([0, 1])\C^2 ([0, 1])
2. there exists an extremal y $\in$ C ([0, 1])\C^1 ([0, 1])
3. every extremal y belongs to C^1 ([0, 1])
4. every extremal y belongs to C^2 ([0, 1])

DECEMBER – 2019

PART-B

27. Let y = ϕ(x) be the extremizing function for the functional

$I(y)=\int_0^1 y^2\left(\frac{dy}{dx}\right)^2 dx,$ subject to

y(0) = 0, y(1) = 1. Then ϕ(1/4) is equal to

1. 1/2
2. 1/4
3. 1/8
4. 1/12

PART-C

28. Let y = y(x) $\in$ C^4([0, 1]) be an extremizing function for the functional

$I(y)=\int_0^1\left[\left(\frac{d^2y}{dx^2}\right)^2-2y\right]dx,$ satisfying

y(0) = 0 = y(1). Then an extremal y(x), satisfying the given conditions at 0 and 1 together with the natural boundary conditions, is given by

1. $\frac{x}{24}(x-1)^3$
2. $\frac{x^2}{24}(x-1)^2$
3. $\frac{x}{24}(x^3-2x^2+1)$
4. $\frac{x}{24}(x^3+x^2-2)$

29. The minimum value of the functional

$I(y)=\int_0^{\pi}\left(\frac{dy}{dx}\right)^2 dx,$

subject to $\int_0^{\pi} y^2(x)\,dx=1,$ y(0) = 0 = y(π) is equal to

1. 1/2
2. 1
3. 2
4. 1/3

JUNE – 20

PART – B

30. The extremal of the functional

$J(y)=\int_0^1[2(y')^2+xy]\,dx,$ y(0) = 0, y(1) = 1, y $\in$ C^2[0, 1]

1. $y=\frac{x^2}{12}+\frac{11x}{12}$
2. $y=\frac{x^3}{3}+\frac{2x^2}{3}$
3. $y=\frac{x^2}{7}+\frac{6x}{7}$
4. $y=\frac{x^3}{24}+\frac{23x}{24}$

PART – C

31. The extremal of the functional

$J(y)=\int_0^1 e^x\sqrt{1+(y')^2}\,dx,\ y\in C^2[0,1]$

is of the form

1. $y=\sec^{-1}\left(\frac{x}{c_1}\right)+c_2,$ where c_1 and c_2 are arbitrary constants
2. $y=\sec^{-1}\left(\frac{x}{c_1}\right)+c_2,$ where $|c_1|<1$ and c_2 is an arbitrary constant

3. $y = \tan^{-1}\left(\frac{x}{c_1}\right) + c_2$, where c_1 and c_2 are arbitrary constants
4. $y = \tan^{-1}\left(\frac{x}{c_1}\right) + c_2$, where $|c_1| > 1$ and c_2 is an arbitrary constant

32. Consider the functional $J(y) = \int_0^{\pi} ((y')^2 - ky^2)\,dx$ with boundary conditions $y(0) = 0$, $y(\pi) = 0$
Which of the following statements are true?
1. It has a unique extremal for all $k \in \mathbb{R}$
2. It has atmost one extremal if $\sqrt{k}$ is not an integer
3. It has infinitely many extremals if $\sqrt{k}$ is an integer
4. It has a unique extremal if $\sqrt{k}$ is an integer

JUNE – 21

PART – B

33. Which of the following is an extremal of the functional $J(y) = \int_{-1}^{1} (y'^2 - 2xy)\,dx$ that satisfy the boundary conditions $y(-1) = -1$ and $y(1) = 1$?

1. $-\frac{x^3}{5} + \frac{6x}{5}$
2. $-\frac{x^5}{8} + \frac{9x}{8}$
3. $-\frac{x^3}{6} + \frac{7x}{6}$
4. $-\frac{x^3}{7} + \frac{8x}{7}$

PART – C

34. Let $X = \{y \in C^1[0, \pi] : y(0) = 0 = y(\pi)\}$ and define $J : X \to \mathbb{R}$ by $J(y) = \int_0^{\pi} y^2(1 - y'^2)dx.$ Which of the following statements are true?
1. $y \equiv 0$ is a local minimum for J with respect to the C^1 norm on X
2. $y \equiv 0$ is a local maximum for J with respect to the C^1 norm on X
3. $y \equiv 0$ is a local minimum for J with respect to the sup norm on X
4. $y \equiv 0$ is a local maximum for J with respect to the sup norm on X

35. Let B be the unit ball in $\mathbb{R}^3$ centered at origin. The Euler-Lagrange equation corresponding to the functional $I(u) = \int_B (1 + |\nabla u|^2)^{\frac{1}{2}} dx$ is

1. $div\left(\frac{\nabla u}{(1 + |\nabla u|^2)^{\frac{1}{2}}}\right) = 0$
2. $\frac{\Delta u}{(1 + |\nabla u|^2)^{\frac{1}{2}}} = 1$
3. $|\nabla u| = 1$
4. $(1 + |\nabla u|^2)\Delta u = \sum_{i,j=1}^{3} u_{x_i} u_{x_j} u_{x_i x_j}$

JUNE – 22

PART – B

36. What is the extremal of the functional $J[y] = \int_{-1}^{0} (12xy - (y')^2)\,dx$ subject to $y(0) = 0$ and $y(-1) = 1$?

1. $y = x^2$
2. $y = \frac{2x^2 + x^4}{3}$
3. $y = -x^3$
4. $y = \frac{x^2 + x^4}{4}$

PART – C

37. Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be two points on the xy-plane with x_1 different from x_2 and $y_1 > y_2$. Consider a curve $C = \{z : z(x_1) = P_1, z(x_2) = P_2\}$. Suppose that a particle is sliding down along the curve C from the point P_1 to P_2 under the influence of gravity. Let T be the time taken to reach point P_2 and g denote the gravitational constant. Which of the following statements are true?

1. $T = \int_{x_1}^{x_2} \sqrt{\frac{1 + (z'(x))^2}{2gz(x)}}\,dx$
2. $T = \int_{x_1}^{x_2} \frac{\sqrt{1 + (z'(x))^2}}{2gz(x)}\,dx$

3. T is minimized when C is a straight line
4. The minimizer of T cannot be a straight line

38. Let X = {u ∈ C^1 [0, 1] : u(0) = u(1) = 0}. Let I : X → R defined as $I(u)=\int_0^1 e^{-u'(t)^2}dt,$ for all u ∈ X. Let $M=\sup_{f\in X} I[f]$ and $m=\inf_{f\in X} I[f]$. Which of the following statements are true?

1. M = 1, m = 0
2. 1 = M > m > 0
3. M is attained
4. m is attained

39. If y(t) is a stationary function of

$$J[y]=\int_{-1}^{1}(1-x^2)(y')^2dx, y(-1)=1, y(1)=1$$

subject to

$$\int_{-1}^{1}y^2=1.$$

Which of the following statements are true?

1. y is unique
2. y is always a polynomial of even order
3. y is always a polynomial of odd order
4. No such y exists

JUNE – 23

PART – B

40. Consider the variational problem (P)

$$J(y(x))=\int_0^1[(y')^2-y|y|y'+xy]dx, \quad y(0)=0,\ y(1)=0.$$

Which of the following statements is correct?

(1) (P) has no stationary function (extremal).
(2) y ≡ 0 is the only stationary function (extremal) for (P).
(3) (P) has a unique stationary function (extremal) y not identically equal to 0.
(4) (P) has infinitely many stationary functions (extremal).

PART – C

41. Let y(x) and z(x) be the stationary functions (extremals) of the variational problem

$$J(y(x),z(x))=\int_0^1[(y')^2+(z')^2+y'z']dx$$

subject to y(0) = 1, y(1) = 0, z(0) = -1, z(1) = 2.
Which of the following statements are correct?

(1) z(x) + 3y(x) = 2 for x ∈ [0, 1].
(2) 3z(x) + y(x) = 2 for x ∈ [0, 1].
(3) y(x) + z(x) = x for x ∈ [0, 1].
(4) y(x) + z(x) = x for x ∈ [0, 1].

42. Suppose y(x) is an extremal of the following functional

$$J(y(x))=\int_0^1(y(x)^2-4y(x)y'(x)+4y'(x)^2)dx$$

subject to y(0) = 1 and y′(0) = 1/2.
Which of the following statements are true?

(1) y is a convex function.
(2) y is concave function.
(3) $y(x_1 + x_2) = y(x_1)\, y(x_2)$ for all x_1, x_2 in [0, 1].
(4) $y(x_1x_2) = y(x_1) + y(x_2)$ for all x_1, x_2 [0, 1].

DECEMBER – 23

PART – B

43. The cardinality of the set of extremals of

$$J[y]=\int_0^1(y')^2dx,$$

subject to

$$y(0)=1,\quad y(1)=6,\quad \int_0^1 y\,dx=3$$

is

(1) 0
(2) 1
(3) 2
(4) countably infinite

PART – C

44. Among the curves connecting the points (1, 2) and (2, 8), let γ be the curve on which an extremal of the functional

$$J[y]=\int_1^2(1+x^3y')\,y'dx$$

can be attained. Then which of the following points lie on the curve γ?

(1) $(\sqrt{2},3)$
(2) $(\sqrt{2},6)$
(3) $\left(\sqrt{3},\frac{22}{3}\right)$
(4) $\left(\sqrt{3},\frac{23}{3}\right)$

45. Define
S = {y ∈ C^1 [0, π] : y(0) = y(π) = 0}

$\|f\|_\infty = \max_{x\in[0,\pi]} |f(x)|$, for all f ∈ S

$B_0(f, \varepsilon) = \{f \in S : \|f\|_\infty < \varepsilon\}$
$B_1(f, \varepsilon) = \{f \in S : \|f\|_\infty + \|f'\|_\infty < \varepsilon\}$

Consider the functional J : S → ℝ given by

$$J[y] = \int_0^\pi (1-(y')^2)y^2 dx.$$

Then there exists ε > 0 such that

(1) $J[y] \le J[0]$, for all $y \in B_0(0, \varepsilon)$
(2) $J[y] \le J[0]$, for all $y \in B_1(0, \varepsilon)$
(3) $J[y] \ge J[0]$, for all $y \in B_0(0, \varepsilon)$
(4) $J[y] \ge J[0]$, for all $y \in B_1(0, \varepsilon)$

JUNE – 24

PART – B

46. Let $B(0, 1) = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ be the open unit disc in $\mathbb{R}^2$, $\partial B(0, 1)$ denote the boundary of B(0, 1), and ν denote unit outward normal to $\partial B(0, 1)$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a given continuous function. The Euler-Lagrange equation of the minimization problem.

$$\min\left\{\frac{1}{2}\iint_{B(0,1)}|\nabla u|^2\,dxdy + \frac{1}{2}\iint_{B(0,1)}e^{u^2}dxdy + \int_{\partial B(0,1)} fuds\right\}$$

subject to $u \in C^1(\overline{B(0,1)})$ is

(1) $\begin{cases}\Delta u = -ue^{u^2} & in\, B(0,1)\\ \dfrac{\partial u}{\partial \nu} = f & on\, \partial B(0,1)\end{cases}$

(2) $\begin{cases}\Delta u = ue^{u^2} + f & in\, B(0,1)\\ u = 0 & on\, \partial B(0,1)\end{cases}$

(3) $\begin{cases}\Delta u = ue^{u^2} & in\, B(0,1)\\ \dfrac{\partial u}{\partial \nu} = -f & on\, \partial B(0,1)\end{cases}$

(4) $\begin{cases}\Delta u = ue^{u^2} & in\, B(0,1)\\ \dfrac{\partial u}{\partial \nu} + u = f & on\, \partial B(0,1)\end{cases}$

PART – C

47. The infimum of the set $\{\int_a^b \sqrt{1+(y'(t))^2}\,dt; y \in C^1[a,b],\ \ y(a) = a^2,\ y(b) = b-5\}$ is

(1) $\dfrac{19\sqrt{2}}{8}$ (2) $19\sqrt{2}$

(3) $\dfrac{19}{8}$ (4) $\dfrac{19}{2\sqrt{2}}$

48. The extremizer of the problem

$$\min\left[\frac{1}{2}\int_{-1}^{1}[(y'(x))^2 + (y(x))^2]dx\right]$$

subject to $y \in C^1[-1, 1]$,

$\int_{-1}^{1} xy(x)dx = 0$ and $y(-1) = y(1) = 1$ is

(1) $\dfrac{e}{1+e^2}(e^x + e^{-x}) + x^2 - 1$

(2) $\dfrac{e}{1+e^2}(e^x + e^{-x}) + 1 - x^2$

(3) $\dfrac{e}{1+e^2}(e^x + e^{-x})$

(4) $\dfrac{e}{1+e^2}(e^x + e^{-x}) + \sin(2\pi x)$

DECEMBER – 2024

PART – B

49. Let S denote the set of all solutions of the Euler-Lagrange equation of the variational problem :

Minimize

$$J[y] = \int_0^1 (y^2 + (y')^2)dx,$$

$$subject\ to\ y(0) = 0, y(1) = 0,\ \int_0^1 y^2 dx = 1.$$

Then the set $\{\varphi\left(\frac{1}{2}\right) : \varphi \in S\}$ is equal to

(1) $\{-\sqrt{2}, \sqrt{2}\}$
(2) $\left\{\frac{\sqrt{2}}{k} : k \in \mathbb{Z}, k \neq 0\right\}$
(3) $\left\{\frac{\sqrt{2}}{k} : k \in \mathbb{N}\right\}$
(4) $\{-\sqrt{2}, 0, \sqrt{2}\}$

PART – C

50. Define

$S := \{y \in C^1[-1,1] : y(-1) = -1, y(1) = 3\}$.

Let φ be the extremal of the functional $J : S \to \mathbb{R}$ given by

$J[y] = \int_{-1}^{1}[(y')^3 + (y')^2]dx.$ Define

$\|y\|_\infty := \max_{x\in[-1,1]} |y(x)|$ for every $y \in S$ and let

$B_0(\varphi,\varepsilon):=\{y \in S : \| y-\varphi \|_\infty < \varepsilon\}, B_1(\varphi,\varepsilon):$
$=\{y \in S : \| y-\varphi \|_\infty + \| y'-\varphi' \|_\infty < \varepsilon\}.$

Then which of the following statements are true?

(1) $\varphi(x) = 2x+1$ for every $x \in [-1,1]$

(2) There exists $\varepsilon > 0$ such that $J[y] \geq J[\varphi]$ for every $y \in B_0(\varphi,\varepsilon)$

(3)There exists $\varepsilon > 0$ such that $J[y] \geq J[\varphi]$ for every $y \in B_1(\varphi,\varepsilon)$

(4)There exists $\varepsilon > 0$ such that $J[y] \leq J[\varphi]$ for every $y \in B_1(\varphi,\varepsilon)$

51. For any b $\in \mathbb{R}$, let S(b) denote the set of all broken extremals with one corner of the variational problem minimize

$J[y] = \int_0^1 ((y')^4 - 3(y')^2)dx,$

$subject\, to\; y(0)=0,\; y(1)=b.$

Then which of the following statements are true?

(1) S(2) has exactly two elements

(2) $S\left(\frac{1}{2}\right)$ has exactly one element

(3) S(2) is empty

(4) $S\left(\frac{1}{2}\right)$ has exactly two elements

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PART – B

52. Let y(x) be the extremal of the functional

$J[y] = \int_0^{\frac{\pi}{4}} ((y')^2 - 4y^2 + 2xy)\, dx$

subject to $y(0)=0, y\left(\frac{\pi}{4}\right)=1.$ Then y(x) is equal to

(1) $\left(1-\frac{\pi}{4}\right)\sin\,(2x)+x$

(2) $\left(1-\frac{\pi}{16}\right)\sin\,(2x)+\frac{x}{4}$

(3) $\left(1+\frac{\pi}{4}\right)\sin\,(2x)-x$

(4) $\left(1+\frac{\pi}{16}\right)\sin\,(2x)-\frac{x}{4}$

PART – C

53. For $\alpha \geq 0,$ consider the functional

$J_\alpha[y] = \int_1^2 \frac{(y')^2}{x^\alpha}dx$

defined for all continuously differentiable functions defined on the interval [1, 2] satisfying the conditions
y(1) = 1, y(2) =2.
Then which of the following statements are true?

1. $y(x) = \frac{1}{15}(x^4+14)$ is an extremal for J_3.
2. $y(x) = \frac{1}{3}(x^2+2)$ is an extremal for J_1.
3. $y(x) = x$ is an extremal for J_0.
4. $y(x) = \frac{1}{2}(x^2-x+2)$ is an extremal for J_1.

54. For a, b, c $\in \mathbb{R}$, consider the variational problem:
Minimize J[y] =

$\int_0^2 [a(y')^2 + 2byy' + cy^2]dx,$

subject to y(0) = 10, y(1) = 100.
Then which of the following statements are true?

1. If (a, b, c) = (-2, 1, -2), then every admissible extremal is minimizer
2. If (a, b, c) = (1, 0, 2), then every admissible extremal is minimizer
3. If (a, b, c) = (2, -1, 1), then every admissible extremal is minimizer
4. If (a, b, c) = (1, -2, 5), then every admissible extremal is minimizer

ANSWERS

1. (3)	2. (2,4)	3. (2)
4. (1,2)	5. (1,2)	6. (3)
7. (2)	8. (2,4)	9. (4)
10. (1,3)	11. (3)	12. (1,3)
13. (2,4)	14. (1)	15. (3)
16. (1,3)	17. (1)	18. (1)
19. (1)	20. (1)	21.
22. (4)	23.	24. (1)
25. (1)	26. (3,4)	27. (1)
28. (3)	29. (2)	30. (4)
31. (1)	32. (2,3)	33. (3)
34. (1)	35. (1,4)	36. (3)
37.	38. (3,4)	39.
40. (3)	41. (1,3)	42. (1,3)
43. (2)	44. (2,3)	45.
46. (3)	47. (1)	48. (3)
49. (4)	50. (1,3)	51. (3,4)
52. (2)	53. (1,2,3)	54. (1,2,3,4)