

COMPLEX ANALYSIS (PYPS)**JUNE – 2014****PART – B**

- Let f, g be meromorphic functions on $\mathbb{C}$. If f has a zero of order k at $z=a$ and g has a pole of order m at $z = 0$, then $g(f(z))$ has
 - a zero of order km at $z=a$
 - a pole of order km at $z=a$
 - a zero of order $|k-m|$ at $z=a$
 - a pole of order $|k-m|$ at $z=a$
- Let $p(x)$ be a polynomial of the real variable x of degree $k \geq 1$. Consider the power series $f(z) = \sum_{n=0}^{\infty} p(n)z^n$, where z is a complex variable. Then the radius of convergence of $f(z)$ is

1. 0	2. 1
3. k	4. ∞

PART – C

- Let f be an entire function. Suppose, for each $a \in \mathbb{R}$, there exists at least one coefficient c_n in $f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n$, which is zero. Then
 - $f^{(n)}(0) = 0$ for infinitely many $n \geq 0$
 - $f^{(2n)}(0) = 0$ for every $n \geq 0$.
 - $f^{(2n+1)}(0) = 0$ for every $n \geq 0$
 - there exists $k \geq 0$ such that $f^{(n)}(0) = 0$ for all $n \geq k$.
- Let $K \subseteq \mathbb{C}$ be a bounded set. Let $H(\mathbb{C})$ denote the set of all entire functions and let $\mathbb{C}(K)$ denote the set of all continuous functions on K . Consider the restriction map $r: H(\mathbb{C}) \rightarrow \mathbb{C}(K)$ given by $r(f) = f|_K$. Then r is injective if

1. K is compact.	2. K is connected.
3. K is uncountable.	4. K is finite.
- For $z \in \mathbb{C}$, define $f(z) = \frac{e^z}{e^z - 1}$. Then
 - f is entire.
 - the only singularities of f are poles.
 - f has infinitely many poles on the imaginary axis.
 - each pole of f is simple.

- Let $D = \{z \in \mathbb{C} : |z| < 1\}$. Then there exists a holomorphic function $f: D \rightarrow \bar{D}$ with $f(0) = 0$ with the property

1. $f'(0) = \frac{1}{2}$	2. $\left f\left(\frac{1}{3}\right) \right = \frac{1}{4}$
3. $f\left(\frac{1}{3}\right) = \frac{1}{2}$	4. $ f'(0) = \sec\left(\frac{\pi}{6}\right)$

DECEMBER – 2014**PART – B**

- Let $p(z) = a_0 + a_1 z + \dots + a_n z^n$ and $q(z) = b_1 z + b_2 z^2 + \dots + b_n z^n$ be complex polynomials. If a_0, b_1 are non-zero complex numbers then the residue of $p(z)/q(z)$ at 0 is equal to

1. $\frac{a_0}{b_1}$	2. $\frac{b_1}{a_0}$
3. $\frac{a_1}{b_1}$	4. $\frac{a_0}{a_1}$
- Let $\sum_{n=1}^{\infty} a_n z^n$ be a convergent power series such that $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = R > 0$. Let p be a polynomial of degree d . Then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n p(n) z^n$ equals

1. R	2. d
3. Rd	4. $R+d$
- Let f be an entire function on $\mathbb{C}$ and let Ω be a bounded open subset of $\mathbb{C}$. Let $S = \{\operatorname{Re} f(z) + \operatorname{Im} f(z) \mid z \in \Omega\}$. Which of the following statements is/are necessarily correct?
 - S is an open set in $\mathbb{R}$
 - S is a closed set in $\mathbb{R}$
 - S is an open set of $\mathbb{C}$
 - S is a discrete set in $\mathbb{R}$

10. Let $u(x+iy)=x^3-3xy^2+2x$. For which of the following functions v , is $u+iv$ a holomorphic function on $\mathbb{C}$?

1. $v(x+iy) = y^3 - 3x^2y + 2y$
2. $v(x+iy) = 3x^2y - y^3 + 2y$
3. $v(x+iy) = x^3 - 3xy^2 + 2x$
4. $v(x+iy) = 0$

PART - C

11. Let f be an entire function on $\mathbb{C}$. Let $g(z) = \overline{f(\bar{z})}$. Which of the following statements is/are correct?

1. if $f(z) \in \mathbb{R}$ for all $z \in \mathbb{R}$ then $f=g$
2. if $f(z) \in \mathbb{R}$ for all $z \in \{z \mid \text{Im } z = 0\} \cup \{z \mid \text{Im } z = a\}$, for some $a > 0$, then $f(z+ia) = f(z-ia)$ for all $z \in \mathbb{C}$.
3. If $f(z) \in \mathbb{R}$ for all $z \in \{z \mid \text{Im } z = 0\} \cup \{z \mid \text{Im } z = a\}$, for some $a > 0$, then $f(z+2ia) = f(z)$ for all $z \in \mathbb{C}$
4. If $f(z) \in \mathbb{R}$ for all $z \in \{z \mid \text{Im } z = 0\} \cup \{z \mid \text{Im } z = a\}$ for some $a > 0$, then $f(z+ia) = f(z)$ all $z \in \mathbb{C}$

12. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function and let r be a positive real number. Then

1. $\sum_{n=0}^{\infty} |a_n|^2 r^{2n} \leq \sup_{|z|=r} |f(z)|^2$
2. $\sup_{|z|=r} |f(z)|^2 \leq \sum_{n=0}^{\infty} |a_n|^2 r^{2n}$
3. $\sum_{n=0}^{\infty} |a_n|^2 r^{2n} \leq \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^2 d\theta$
4. $\sup_{|z|=r} |f(z)|^2 \leq \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^2 d\theta$

JUNE - 2015

PART - B

13. $\int_{|z+1|=2} \frac{z^2}{4-z^2} dz =$

1. 0
2. $-2\pi i$
3. $2\pi i$
4. 1

14. How many elements does the set $\{z \in \mathbb{C} / z^{60} = -1, z^k \neq -1 \text{ for } 0 < k < 60\}$ have?

1. 24
2. 30
3. 32
4. 45

15. Let f be a real valued harmonic function on $\mathbb{C}$, that is, f satisfies the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

Define the functions

$$g = \frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y}, \quad h = \frac{\partial f}{\partial x} + i \frac{\partial f}{\partial y}$$

Then

1. g and h are both holomorphic functions.
2. g is holomorphic, but h need not be holomorphic.
3. h is holomorphic, but g need not be holomorphic.
4. both g and h are identically equal to the zero function.

PART - C

16. Let f be an entire function. Which of the following statements are correct?

1. f is constant if the range of f is contained in a straight line.
2. f is constant if f has uncountably many zeros.
3. f is constant if f is bounded on $\{z \in \mathbb{C} : \text{Re}(z) \leq 0\}$.
4. f is constant if the real part of f is bounded.

17. Let f be analytic function defined on the open unit disc in $\mathbb{C}$. Then f is constant if

1. $f\left(\frac{1}{n}\right) = 0$ for all $n \geq 1$.
2. $f(z) = 0$ for all $|z| = \frac{1}{2}$.
3. $f\left(\frac{1}{n^2}\right) = 0$ for all $n \geq 1$.
4. $f(z) = 0$ for all $z \in (-1, 1)$.

18. Let p be a polynomial in 1-complex variable. Suppose all zeroes of p are in the upper half plane $H = \{z \in \mathbb{C} \mid \text{Im}(z) > 0\}$. Then

1. $\text{Im} \frac{p'(z)}{p(z)} > 0$ for $z \in \mathbb{R}$.

$$2. \operatorname{Re} i \frac{p'(z)}{p(z)} < 0 \text{ for } z \in \mathbb{R}$$

$$3. \operatorname{Im} \frac{p'(z)}{p(z)} > 0 \text{ for } z \in \mathbb{C}, \text{ with } \operatorname{Im} z < 0$$

$$4. \operatorname{Im} \frac{p'(z)}{p(z)} > 0 \text{ for } z \in \mathbb{C} \text{ with } \operatorname{Im} z > 0$$

$$2. a_n = (-1)^{n-1} \frac{1}{n}$$

$$3. a_n = \frac{1}{2n}$$

$$4. a_n = n \text{ if } 4 \text{ does not divide } n \text{ and } a_n = \frac{1}{n} \text{ if } 4 \text{ divides } n$$

DECEMBER – 2015**PART – B**

19. Let $a, b, c, d \in \mathbb{R}$ be such that $ad - bc > 0$. Consider the Mobius transformation

$$T_{a,b,c,d}(z) = \frac{az + b}{cz + d}. \text{ Define}$$

$$H_+ = \{z \in \mathbb{C} : \operatorname{Im}(z) > 0\}, H_- = \{z \in \mathbb{C} : \operatorname{Im}(z) < 0\}$$

$$R_+ = \{z \in \mathbb{C} : \operatorname{Re}(z) > 0\}, R_- = \{z \in \mathbb{C} : \operatorname{Re}(z) < 0\}.$$

Then, $T_{a,b,c,d}$ maps

- | | |
|---------------------|---------------------|
| 1. H_+ to H_+ . | 2. H_+ to H_- . |
| 3. R_+ to R_+ . | 4. R_+ to R_- . |

20. What is the cardinality of the set

$$\{z \in \mathbb{C} \mid z^{98} = 1 \text{ and } z^n \neq 1 \text{ for any } 0 < n < 98\}?$$

- | | |
|--------|--------|
| 1. 0. | 2. 12. |
| 3. 42. | 4. 49. |

21. Consider the following power series in the complex variable z :

$$f(z) = \sum_{n=1}^{\infty} n \log n z^n, g(z) =$$

$$\sum_{n=1}^{\infty} \frac{e^{n^2}}{n} z^n. \text{ If } r, R \text{ are the radii of}$$

convergence of f and g respectively, then

- | | |
|--------------------------|--------------------------|
| 1. $r = 0, R = 1$. | 2. $r = 1, R = 0$. |
| 3. $r = 1, R = \infty$. | 4. $r = \infty, R = 1$. |

PART – C

22. Let $f(z) = \frac{1}{e^z - 1}$ for all $z \in \mathbb{C}$ such that

$e^z \neq 1$. Then

- f is meromorphic.
- the only singularities of f are poles.
- f has infinitely many poles on the imaginary axis.
- Each pole of f is simple.

23. Let f be an analytic function in $\mathbb{C}$. Then f is constant if the zero set of f contains the sequence

- $a_n = 1/n$

24. Consider the function $f(z) = \frac{1}{z}$ on the

$$\text{annulus } A = \left\{ z \in \mathbb{C} : \frac{1}{2} < |z| < 2 \right\}.$$

Which of the following is/are true?

- There is a sequence $\{p_n(z)\}$ of polynomials that approximate $f(z)$ uniformly on compact subsets of A .
- There is a sequence $\{r_n(z)\}$ of rational functions, whose poles are contained in $\mathbb{C} \setminus A$ and which approximates $f(z)$ uniformly on compact subsets of A .
- No sequence $\{p_n(z)\}$ of polynomials approximate $f(z)$ uniformly on compact subsets of A .
- No sequence $\{r_n(z)\}$ of rational functions whose poles are contained in $\mathbb{C} \setminus A$, approximate $f(z)$ uniformly on compact subsets of A .

JUNE – 2016**PART – B**

25. Let $P(z), Q(z)$ be two complex non-constant polynomials of degree m, n respectively. The number of roots of $P(z) = P(z)Q(z)$ counted with multiplicity is equal to:

- | | |
|-------------------|-------------------|
| 1. $\min\{m, n\}$ | 2. $\max\{m, n\}$ |
| 3. $m+n$ | 4. $m-n$ |

26. Let D be the open unit disc in $\mathbb{C}$ and $H(D)$ be the collection of all holomorphic functions on it.

$$\text{Let } S = \left\{ f \in H(D) : f\left(\frac{1}{2}\right) = \frac{1}{2}, f\left(\frac{1}{4}\right) \right.$$

$$\left. = \frac{1}{4}, \dots, f\left(\frac{1}{2n}\right) = \frac{1}{2n}, \dots \right\} \text{ and}$$

$$T = \left\{ f \in H(D) : f\left(\frac{1}{2}\right) = f\left(\frac{1}{3}\right) = \frac{1}{2}, f\left(\frac{1}{4}\right) \right.$$

$$\left. = f\left(\frac{1}{5}\right) = \frac{1}{4}, \dots, f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{2n}, \dots \right\} \text{ Then}$$

- Both S, T are singleton sets
- S is a singleton set but $T = \emptyset$
- T is a singleton set but $S = \emptyset$

4. Both S,T are empty

27. Let $P(x)$ be a polynomial of degree $d \geq 2$. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} p(n)z^n \text{ is}$$

- | | |
|-------------|-------------------|
| 1. 0 | 2. 1 |
| 3. ∞ | 4. dependent on d |

28. The residue of the function $f(z) = e^{-e^{1/z}}$ at $z=0$ is:

- | | |
|-----------------|-----------------|
| 1. $1 + e^{-1}$ | 2. e^{-1} |
| 3. $-e^{-1}$ | 4. $1 - e^{-1}$ |

PART - C

29. Let $H = \{z = x + iy \in \mathbb{C} : y > 0\}$ be the upper half plane and $D = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc. Suppose that f is a Mobius transformation, which maps H conformally onto D . Suppose that $f(2i) = 0$. Pick each correct statement from below.

1. f has a simple pole at $z = -2i$.
2. f satisfies $f(i) \overline{f(-i)} = 1$.
3. f has an essential singularity at $z = -2i$.
4. $|f(2 + 2i)| = \frac{1}{\sqrt{5}}$.

30. Consider the function $F(z) = \int_1^2 \frac{1}{(x-z)^2} dx$,

$\text{Im}(z) > 0$. Then there is a meromorphic function $G(z)$ on $\mathbb{C}$ that agrees with $F(z)$ when $\text{Im}(z) > 0$, such that

1. $1, \infty$ are poles of $G(z)$
2. $0, 1, \infty$ are poles of $G(z)$
3. $1, 2$ are poles of $G(z)$
4. $1, 2$ are simple poles of $G(z)$.

31. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an entire function. Suppose that $f = u + iv$ where u, v are the real and imaginary parts of f respectively. Then f is constant if

1. $\{u(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded
2. $\{v(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded
3. $\{u(x, y) + v(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded
4. $\{u^2(x, y) + v^2(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded

32. Let $A = \{z \in \mathbb{C} \mid |z| > 1\}$, $B = \{z \in \mathbb{C} \mid z \neq 0\}$. Which of the following are true?

1. There is a continuous onto function $f: A \rightarrow B$
2. There is a continuous one to one function $f: B \rightarrow A$
3. There is a nonconstant analytic function $f: B \rightarrow A$
4. There is a nonconstant analytic function $f: A \rightarrow B$

DECEMBER - 2016

PART - B

33. The radius of convergence of the series

$$\sum_{n=1}^{\infty} z^{n^2} \text{ is}$$

- | | |
|------|-------------|
| 1. 0 | 2. ∞ |
| 3. 1 | 4. 2 |

34. Let C be the circle $|z| = 3/2$ in the complex plane that is oriented in the counter clockwise direction. The value of a for which

$$\int_C \left(\frac{z+1}{z^2-3z+2} + \frac{a}{z-1} \right) dz = 0 \text{ is}$$

- | | |
|------|-------|
| 1. 1 | 2. -1 |
| 3. 2 | 4. -2 |

35. Suppose f and g are entire functions and $g(z) \neq 0$ for all $z \in \mathbb{C}$. If $|f(z)| \leq |g(z)|$, then we conclude that

1. $f(z) \neq 0$ for all $z \in \mathbb{C}$.
2. f is a constant function.
3. $f(0) = 0$.
4. for some $C \in \mathbb{C}$, $f(z) = Cg(z)$.

36. Let f be a holomorphic function on $0 < |z| < \epsilon$, $\epsilon > 0$ given by a convergent Laurent series

$$\sum_{n=-\infty}^{\infty} a_n z^n. \text{ Given also that } \lim_{z \rightarrow 0} |f(z)| = \infty,$$

We can conclude that

1. $a_1 \neq 0$ and $a_n = 0$ for all $n \geq 2$
2. $a_N \neq 0$ for some $N \geq 1$ and $a_n = 0$ for all $n > N$
3. $a_n = 0$ for all $n \geq 1$
4. $a_n \neq 0$ for all $n \geq 1$

PART - C

37. Let $f(z)$ be the meromorphic function given by $\frac{z}{(1-e^z)\sin z}$. Then

1. $z=0$ is a pole of order 2.

2. for every $k \in \mathbb{Z}$, $z = 2\pi i k$ is a simple pole.
3. for every $k \in \mathbb{Z} \setminus \{0\}$, $z = k\pi$ is a simple pole.
4. $z = \pi + 2\pi i$ is a pole.

38. Consider the polynomial

$$P(z) = \sum_{n=1}^N a_n z^n, 1 \leq N < \infty, a_n \in \mathbb{R} \setminus \{0\}. \text{ Then,}$$

with $D = \{w \in \mathbb{C} : |w| < 1\}$

1. $P(D) \subseteq \mathbb{R}$
2. $P(D)$ is open
3. $P(D)$ is closed
4. $P(D)$ is bounded

39. Consider the polynomial

$$P(z) = \left(\sum_{n=0}^5 a_n z^n \right) \left(\sum_{n=0}^9 b_n z^n \right) \text{ where } a_n, b_n \in \mathbb{R}$$

$\forall n, a_5 \neq 0, b_9 \neq 0$. Then counting roots with multiplicity we can conclude that $P(z)$ has

1. at least two real roots.
2. 14 complex roots
3. no real roots
4. 12 complex roots.

40. Let D be the open unit disc in $\mathbb{C}$. Let $g: D \rightarrow D$ be holomorphic, $g(0)=0$, and let

$$h(z) = \begin{cases} \frac{g(z)}{z}, & z \in D, z \neq 0 \\ g'(0), & z = 0 \end{cases}. \text{ Which of the}$$

following statements are true?

1. h is holomorphic in D .
2. $h(D) \subseteq \overline{D}$.
3. $|g'(0)| > 1$
4. $\left| g\left(\frac{1}{2}\right) \right| \leq \frac{1}{2}$

JUNE - 2017

PART - B

41. Let C denote the unit circle centered at the origin in $\mathbb{C}$. Then $\frac{1}{2\pi i} \int_C |1+z+z^2|^2 dz$, where

the integral is taken anti-clockwise along C , equals

1. 0
2. 1
3. 2
4. 3

42. Consider the power series

$$f(x) = \sum_{n=2}^{\infty} \log(n) x^n.$$

The radius of convergence of the series $f(x)$ is

1. 0
2. 1
3. 3
4. ∞

43. For an odd integer $k \geq 1$, let $\mathcal{F}$ be the set of all entire functions f such that

$$f(x) = |x^k| \text{ for all } x \in (-1, 1). \text{ Then the}$$

cardinality of $\mathcal{F}$ is

1. 0
2. 1
3. strictly greater than 1 but finite
4. infinite

44. Suppose f is holomorphic in an open neighbourhood of $z_0 \in \mathbb{C}$. Given that the

series $\sum_{n=0}^{\infty} f^{(n)}(z_0)$ converges absolutely, we

can conclude that

1. f is constant
2. f is a polynomial
3. f can be extended to an entire function
4. $f(x) \in \mathbb{R}$ for all $x \in \mathbb{R}$

PART - C

45. Let $f = u + iv$ be an entire function where u, v are the real and imaginary parts of f respectively. If the Jacobian matrix

$$J_a = \begin{bmatrix} u_x(a) & u_y(a) \\ v_x(a) & v_y(a) \end{bmatrix} \text{ is symmetric for all } a \in \mathbb{C}, \text{ then}$$

1. f is a polynomial.
2. f is a polynomial of degree ≤ 1 .
3. f is necessarily a constant function
4. f is a polynomial of degree strictly greater than 1.

46. Consider the function $f(z) = \frac{\sin(\pi z/2)}{\sin(\pi z)}$.

Then f has poles at

1. all integers
2. all even integers
3. all odd integers
4. all integers of the form $4k+1, k \in \mathbb{Z}$

47. Consider the Möbius transformation

$$f(z) = \frac{1}{z}, z \in \mathbb{C}, z \neq 0. \text{ If } C \text{ denotes a circle}$$

with positive radius passing through the origin, then f map $C \setminus \{0\}$ to

1. a circle
2. a line
3. a line passing through the origin
4. a line not passing through the origin

48. For which among the following functions $f(z)$ defined on $G = \mathbb{C} \setminus \{0\}$, is there no sequence of polynomials approximating $f(z)$ uniformly on compact subsets of G ?

1. $\exp(z)$
2. $1/z$
3. z^2
4. $1/z^2$

DECEMBER – 2017

PART – B

49. The function $f : \mathbb{C} \rightarrow \mathbb{C}$ defined by $f(z) = e^z + e^{-z}$ has
1. finitely many zeros
 2. no zeros
 3. only real zeros
 4. has infinitely many zeros

50. Let f be a holomorphic function in the open unit disc such that $\lim_{z \rightarrow 1} f(z)$ does not exist. Let $\sum_{n=0}^{\infty} a_n z^n$ be the Taylor series of f about $z = 0$ and let R be its radius of convergence. Then

1. $R = 0$
2. $0 < R < 1$
3. $R = 1$
4. $R > 1$

51. Let C be the circle of radius 2 with centre at the origin in the complex plane, oriented in the anti-clockwise direction. Then the integral

$$\oint_C \frac{dz}{(z-1)^2}$$
 is equal to

1. $\frac{1}{2\pi i}$
2. $2\pi i$
3. 1
4. 0

52. Let D be the open unit disc in the complex plane and $U = D \setminus \left\{ -\frac{1}{2}, \frac{1}{2} \right\}$. Also, let

$H_1 = \{f : D \rightarrow \mathbb{C} \mid f \text{ is holomorphic and bounded}\}$
 and $H_2 = \{f : U \rightarrow \mathbb{C} \mid f \text{ is holomorphic and bounded}\}$. Then the map $r : H_1 \rightarrow H_2$ given by $r(f) = f|_U$, the restriction of f to U , is

1. injective but not surjective
2. surjective but not injective
3. injective and surjective
4. neither injective nor surjective

PART – C

53. Let f be an entire function. Consider $A = \{z \in \mathbb{C} \mid f^{(n)}(z) = 0 \text{ for some positive integer } n\}$. Then

1. if $A = \mathbb{C}$, then f is a polynomial
2. if $A = \mathbb{C}$, then f is a constant function
3. if A is uncountable, then f is a polynomial
4. If A is uncountable, then f is a constant function

54. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be a holomorphic function and let u be the real part of f and v the imaginary part of f . Then, for $x, y \in \mathbb{R}$, $|f'(x + iy)|^2$ is equal to

1. $u_x^2 + u_y^2$
2. $u_x^2 + v_x^2$
3. $v_y^2 + u_y^2$
4. $v_x^2 + v_y^2$

55. Let $p(z) = z^n + a_{n-1}z^{n-1} + \dots + a_0$, where $a_0, \dots, a_{n-1}$ are complex numbers and let $q(z) = 1 + a_{n-1}z + \dots + a_0z^n$. If $|p(z)| \leq 1$ for all z with $|z| \leq 1$ then

1. $|q(z)| \leq 1$ for all z with $|z| \leq 1$
2. $q(z)$ is a constant polynomial
3. $p(z) = z^n$ for all complex numbers z
4. $p(z)$ is a constant polynomial

56. Let f be a non-constant entire function and let E be the image of f . Then

1. E is an open set
2. $E \cap \{z : |z| < 1\}$ is empty
3. $E \cap \mathbb{R}$ is non – empty
4. E is a bounded set

JUNE – 2018

PART – B

57. The value of the integral $\oint_{|1-z|=1} \frac{e^z}{z^2-1} dz$ is

1. 0
2. $(\pi i)e$
3. $(\pi i)e - (\pi i)e^{-1}$
4. $(e + e^{-1})$

58. Let $f : \{z \mid |z| < 1\} \rightarrow \mathbb{C}$ be a non-constant analytic function. Which of the following conditions can possibly be satisfied by f ?

1. $f\left(\frac{1}{n}\right) = f\left(\frac{-1}{n}\right) = \frac{1}{n^2} \quad \forall n \in \mathbb{N}$
2. $f\left(\frac{1}{n}\right) = f\left(\frac{-1}{n}\right) = \frac{1}{2n+1} \quad \forall n \in \mathbb{N}$
3. $\left|f\left(\frac{1}{n}\right)\right| < 2^{-n} \quad \forall n \in \mathbb{N}$
4. $\frac{1}{\sqrt{n}} < \left|f\left(\frac{1}{n}\right)\right| < \frac{2}{\sqrt{n}} \quad \forall n \in \mathbb{N}$

59. Consider the map $\varphi: \mathbb{C} \setminus \{1\} \rightarrow \mathbb{C}$ given by

$$\varphi(z) = \frac{1+z}{1-z}. \text{ Which of the following is true?}$$

1. $\varphi(\{z \in \mathbb{C} \mid |z| < 1\}) \subseteq \{z \in \mathbb{C} \mid |z| < 1\}$
2. $\varphi(\{z \in \mathbb{C} \mid \operatorname{Re}(z) < 0\}) \subseteq \{z \in \mathbb{C} \mid \operatorname{Re}(z) < 0\}$
3. φ is onto
4. $\varphi(\mathbb{C} \setminus \{1\}) = \mathbb{C} \setminus \{-1\}$

60. Suppose that f is a non constant analytic function defined over $\mathbb{C}$. Then which of the following is false?

1. f is unbounded
2. f sends open sets into open sets
3. there exists an open connected domain U on which f is never $f|_U$ attains its minimum at one point of U
4. the image of f is dense in $\mathbb{C}$

PART - C

61. Let Ω be an open connected subset of $\mathbb{C}$. Let $E = \{z_1, z_2, \dots, z_j\} \subseteq \Omega$. Suppose that $f: \Omega \rightarrow \mathbb{C}$ is a function such that $f|_{(\Omega \setminus E)}$ is analytic. Then f is analytic on Ω if

1. f is continuous on Ω
2. f is bounded on Ω
3. for every j , if $\sum_{m \in \mathbb{Z}} a_m (z - z_j)^m$ is Laurent series expansion of f at z_j , then $a_m = 0$ for $m = -1, -2, -3, \dots$
4. for every j , if $\sum_{m \in \mathbb{Z}} a_m (z - z_j)^m$ is Laurent series expansion of f at z_j , then $a_{-1} = 0$

62. Suppose that $f: \mathbb{C} \rightarrow \mathbb{C}$ is an analytic function. Then f is a polynomial if

1. for any point $a \in \mathbb{C}$, if $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ is a power series expansion at a , then $a_n = 0$ for at least one n
2. $\lim_{|z| \rightarrow \infty} |f(z)| = \infty$
3. $\lim_{|z| \rightarrow \infty} |f(z)| = M$ for some M
4. $|f(z)| \leq M|z|^n$ for $|z|$ sufficiently large and for some n

63. Let D be the open unit disk centered at 0 in $\mathbb{C}$ and $f: D \rightarrow \mathbb{C}$ be an analytic function. Let $f = u + iv$, where u, v are the real and imaginary

parts of f . If $f(z) = \sum a_n z^n$ is the power series of f , then f is constant if

1. $\bar{f}$ is analytic
2. $u(1/2) \geq u(z) \quad \forall z \in D$
3. The set $\{n \in \mathbb{N} \mid a_n = 0\}$ is infinite
4. For any closed curve γ in D ,
$$\int_{\gamma} \frac{f(z) dz}{(z-a)^2} = 0 \quad \forall a \in D \text{ with } |a| \geq \frac{1}{2}$$

64. Which of the following statements are true?

1. If $\{a_k\}$ is bounded then $\sum_{k=0}^{\infty} a_k z^k$ defines an analytic function on the open unit disk
2. If $\sum_{k=0}^{\infty} a_k z^k$ defines an analytic function on the open disk then $\{a_k\}$ must converge to zero
3. If $f(z) = \sum_{k=0}^{\infty} a_k z^k$ and $g(z) = \sum_{k=0}^{\infty} b_k z^k$ are two power series functions whose radii of convergence are 1, then the product $f.g$ has a power series representation of the form $\sum_{k=0}^{\infty} c_k z^k$ on the open unit disk
4. If $f(z) = \sum_{k=0}^{\infty} a_k z^k$ has a radius of convergence 1, then f is continuous on $\Omega = \{z \in \mathbb{C} \mid |z| \leq 1\}$

DECEMBER - 2018

PART - B

65. Consider the polynomials $p(z), q(z)$ in the complex variable z and let $I_{p,q} = \oint_{\gamma} p(z) \overline{q(z)} dz$, where γ denotes the

closed contour $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$. Then

1. $I_{z^m, z^n} = 0$ for all positive integers m, n with $m \neq n$
2. $I_{z^n, z^n} = 2\pi i$ for all positive integers n
3. $I_{p,1} = 0$ for all polynomials p
4. $I_{p,q} = p(0) \overline{q(0)}$ for all polynomials p, q

66. Let $\gamma(t) = 3e^{it}, 0 \leq t \leq 2\pi$ be the positively oriented circle of radius 3 centred at the origin. The value of λ for which

$$\oint_{\gamma} \frac{\lambda}{z-2} dz = \oint_{\gamma} \frac{1}{z^2 - 5z + 4} dz \text{ is}$$

1. $\lambda = \frac{-1}{3}$
2. $\lambda = 0$
3. $\lambda = \frac{1}{3}$
4. $\lambda = 1$

2. $\tan z$ is a meromorphic function on $\mathbb{C}$
3. $\tan z$ has an isolated singularity at ∞
4. $\tan z$ has a non-isolated singularity at ∞

JUNE – 2019

- 67.** Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be a non-constant entire function and let $\text{Image}(f) = \{w \in \mathbb{C} : \exists z \in \mathbb{C} \text{ such that } f(z) = w\}$. Then
1. The interior of $\text{Image}(f)$ is empty
 2. $\text{Image}(f)$ intersects every line passing through the origin
 3. There exists a disc in the complex plane, which is disjoint from $\text{Image}(f)$
 4. $\text{Image}(f)$ contains all its limit points

PART - C

- 68.** Let H denote the upper half plane, that is, $H = \{z = x + iy : y > 0\}$. For $z \in H$, which of the following are true?

1. $\frac{1}{z} \in H$
2. $\frac{1}{z^2} \in H$
3. $\frac{-z}{z+1} \in H$
4. $\frac{z}{2z+1} \in H$

- 69.** Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be an analytic function. Then which of the following statements are true?

1. If $|f(z)| \leq 1$ for all $z \in \mathbb{C}$, then f' has infinitely many zeroes in $\mathbb{C}$
2. If f is onto, then the function $f(\cos z)$ is onto
3. If f is onto, then the function $f(e^z)$ is onto
4. If f is one-one, then the function $f(z^4 + z + 2)$ is one-one

- 70.** Consider the entire functions $f(z) = 1 + z + z^{20}$ and $g(z) = e^z$, $z \in \mathbb{C}$. Which of the following statements are true?

1. $\lim_{|z| \rightarrow \infty} |f(z)| = \infty$
2. $\lim_{|z| \rightarrow \infty} |g(z)| = \infty$
3. $f^{-1}(\{z \in \mathbb{C} : |z| \leq R\})$ is bounded for every $R > 0$
4. $g^{-1}(\{z \in \mathbb{C} : |z| \leq R\})$ is bounded for every $R > 0$

- 71.** Which of the following statements are true?
1. $\tan z$ is an entire function

PART - B

- 72.** Let C be the counter-clockwise oriented circle of radius $\frac{1}{2}$ centred at $i = \sqrt{-1}$. Then

the value of the contour integral $\oint_C \frac{dx}{x^4 - 1}$ is

1. $-\pi/2$
2. $\pi/2$
3. $-\pi$
4. π

- 73.** Consider the function $f: \mathbb{C} \rightarrow \mathbb{C}$ given by $f(z) = e^z$. Which of the following is false?

1. $f(\{z \in \mathbb{C} : |z| < 1\})$ is not an open set
2. $f(\{z \in \mathbb{C} : |z| \leq 1\})$ is not an open set
3. $f(\{z \in \mathbb{C} : |z| = 1\})$ is a closed set
4. $f(\{z \in \mathbb{C} : |z| > 1\})$ is an unbounded open set

- 74.** Given a real number $a > 0$, consider the triangle Δ with vertices $0, a, a + ia$. If Δ is given the counter clockwise orientation, then the contour integral $\oint_{\Delta} \text{Re}(z) dz$ (with $\text{Re}(z)$ denoting the real part of z) is equal to

1. 0
2. $i \frac{a^2}{2}$
3. ia^2
4. $i \frac{3a^2}{2}$

- 75.** Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be an entire function such that $\lim_{x \rightarrow 0} \left| f\left(\frac{1}{x}\right) \right| = \infty$. Then which of the following is true?

1. f is constant
2. f can have infinitely many zeros
3. f can have at most finitely many zeros
4. f is necessarily nowhere vanishing

PART - C

- 76.** Let $f(z) = (z^3 + 1) \sin z^2$ for $z \in \mathbb{C}$. Let $f(z) = u(x, y) + i v(x, y)$, where $z = x + iy$ and u, v

are real valued functions. Then which of the following are true?

1. $u : \mathbb{R}^2 \rightarrow \mathbb{R}$ is infinitely differentiable
2. u is continuous but need not be differentiable
3. u is bounded
4. f can be represented by an absolutely convergent power series $\sum_{n=0}^{\infty} a_n z^n$ for all $z \in \mathbb{C}$

77. Let $\operatorname{Re}(z)$, $\operatorname{Im}(z)$ denote the real and imaginary parts of $z \in \mathbb{C}$, respectively. Consider the domain

$\Omega = \{z \in \mathbb{C} : \operatorname{Re}(z) > |\operatorname{Im}(z)|\}$ and let $f_n(z) = \log z^n$, where $n \in \{1, 2, 3, 4\}$ and where $\log : \mathbb{C} \setminus (-\infty, 0] \rightarrow \mathbb{C}$ defines the principal branch of logarithm. Then which of the following are true?

1. $f_1(\Omega) = \{z \in \mathbb{C} : 0 \leq |\operatorname{Im}(z)| < \pi/4\}$
2. $f_2(\Omega) = \{z \in \mathbb{C} : 0 \leq |\operatorname{Im}(z)| < \pi/2\}$
3. $f_3(\Omega) = \{z \in \mathbb{C} : 0 \leq |\operatorname{Im}(z)| < 3\pi/4\}$
4. $f_4(\Omega) = \{z \in \mathbb{C} : 0 \leq |\operatorname{Im}(z)| < \pi\}$

78. Consider the set

$F = \{f : \mathbb{C} \rightarrow \mathbb{C} \mid f \text{ is an entire function, } |f'(z)| \leq |f(z)| \text{ for all } z \in \mathbb{C}\}$.

Then which of the following are true?

1. F is a finite set
2. F is an infinite set
3. $F = \{\beta e^{\alpha z} : \beta \in \mathbb{C}, \alpha \in \mathbb{C}\}$
4. $F = \{\beta e^{\alpha z} : \beta \in \mathbb{C}, |\alpha| \leq 1\}$

79. Let $D = \{z \in \mathbb{C} \mid |z| < 1\}$ and $\omega \in D$. Define

$F_\omega : D \rightarrow D$ by $F_\omega(z) = \frac{\omega - z}{1 - \bar{\omega}z}$. Then which

of the following are true?

1. F is one to one
2. F is not one to one
3. F is onto
4. F is not onto

DECEMBER – 2019

PART – B

80. For $z \in \mathbb{C}$, let $f(z) = \begin{cases} \frac{\bar{z}^2}{z} & \text{if } z \neq 0. \\ 0 & \text{otherwise.} \end{cases}$

Then which of the following statements is false?

1. $f(z)$ is continuous everywhere

2. $f(z)$ is not analytic in any open neighbourhood of zero
3. $zf(z)$ satisfies the Cauchy-Riemann equations at zero
4. $f(z)$ is analytic in some open subset of $\mathbb{C}$

81. Let $T : \mathbb{C} \rightarrow M_2(\mathbb{R})$ be the map given by

$$T(z) = T(x + iy) = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$

Then which of the following statements is false?

1. $T(z_1 z_2) = T(z_1) T(z_2)$ for all $z_1, z_2 \in \mathbb{C}$
2. $T(z)$ is singular if and only if $z = 0$
3. There does not exist non-zero $A \in M_2(\mathbb{R})$ such that the trace of $T(z)A$ is zero for all $z \in \mathbb{C}$
4. $T(z_1 + z_2) = T(z_1) + T(z_2)$ for all $z_1, z_2 \in \mathbb{C}$

82. Consider the polynomial $f(z) = z^2 + az + p^{11}$, where $a \in \mathbb{Z} \setminus \{0\}$ and $p \geq 13$ is a prime. Suppose that $a^2 \leq 4p^{11}$. Which of the following statements is true?

1. f has a zero on the imaginary axis
2. f has a zero for which the real and imaginary parts are equal
3. f has distinct roots
4. f has exactly one real root

83. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an entire function with

$$f\left(\frac{1}{n}\right) = \frac{1}{n^2} \text{ for all } n \in \mathbb{N}.$$

Then which of the following statements is true?

1. No such f exists
2. such an f is not unique
3. $f(z) = z^2$ for all $z \in \mathbb{C}$
4. f need not be a polynomial function

PART – C

84. Let U be an open subset of $\mathbb{C}$ and $f : U \rightarrow \mathbb{C}$ be an analytic function. Then which of the following are true?

1. If f is one-one, then $f(U)$ is open in $\mathbb{C}$

2. If f is onto, then $U = \mathbb{C}$
 3. If f is onto, then f is one-one
 4. $f(U)$ is closed in $\mathbb{C}$, then $f(U)$ is connected

85. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an analytic function. For $z_0 \in \mathbb{C}$, which of the following statements are true?

1. can take the value z_0 at finitely many points in $\frac{1}{n} \mid n \in \mathbb{N}$
 2. $f(1/n) = z_0$ for all $n \in \mathbb{N} \Rightarrow f$ is the constant function z_0
 3. $f(n) = z_0$ for all $n \in \mathbb{N} \Rightarrow f$ is the constant function z_0
 4. $f(r) = z_0$ for all $r \in \mathbb{Q} \cap [1, 2] \Rightarrow f$ is the constant function z_0

86. Let $U \subset \mathbb{C}$ be an open connected set and $f : U \rightarrow \mathbb{C}$ be a non-constant analytic function. Consider the following two sets:

$$X = \{z \in U : f(z) = 0\}$$

$Y = \{z \in U : f \text{ vanishes on an open neighbourhood of } z \text{ in } U\}$

Then which of the following statements are true?

1. X is closed in U
 2. Y is closed in U
 3. X has empty interior
 4. Y is open in U

87. Consider the power series

$$f(z) = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n)!}. \text{ Which of the}$$

following are true?

1. Radius of convergence of $f(z)$ is infinite
 2. The set $\{f(x) : x \in \mathbb{R}\}$ is bounded
 3. The set $\{f(x) : -1 < x < 1\}$ is bounded
 4. $f(z)$ has infinitely many zeroes

JUNE – 2020

PART – B

88. Let γ be the positively oriented circle in the complex plane given by

$$\{z \in \mathbb{C} : |z - 1| = 1\}.$$

Then $\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z^3 - 1}$ equals

1. 3
 2. 1/3
 3. 2
 4. 1/2

89. For a positive integer p , consider the holomorphic function

$$f(z) = \frac{\sin z}{z^p} \text{ for } z \in \mathbb{C} \setminus \{0\}.$$

For which values of p does there exist a holomorphic function $g : \mathbb{C} \setminus \{0\} \rightarrow \mathbb{C}$ such that $f(z) = g'(z)$ for $z \in \mathbb{C} \setminus \{0\}$?

1. All even integers
 2. All odd integers
 3. All multiples of 3
 4. All multiples of 4

90. Let γ be the positively oriented circle in the complex plane given by $\{z \in \mathbb{C} : |z - 1| =$

$1/2\}$. The line integral $\int_{\gamma} \frac{ze^{1/z}}{z^2 - 1} dz$ equals

1. $i\pi e$
 2. $-i\pi e$
 3. πe
 4. $-\pi e$

91. Let p be a positive integer. Consider the closed curve $r(t) = e^{it}$, $0 \leq t < 2\pi$. Let f be a function holomorphic in $\{z : |z| < R\}$ where $R > 1$. If f has a zero only at z_0 , $0 < |z_0| < R$, and it is of multiplicity q , then

$$\frac{1}{2\pi i} \int_{\gamma} \frac{f'(z)}{f(z)} z^p dz \text{ equals}$$

1. qz_0^p
 2. $z_0 q^p$
 3. pz_0^q
 4. $z_0 p^q$

PART – C

92. For $z \neq -i$, let $f(z) = \exp\left(\frac{1}{z+i}\right) - 1$. Which of the following are true?

1. f has finitely many zeroes
 2. f has a sequence of zeroes that converges to a removable singularity of f
 3. f has a sequence of zeroes that converges to a pole of f
 4. f has a sequence of zeroes that converges to an essential singularity of f

93. Let f be a holomorphic function on the open unit disc

$\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. Suppose that $|f| \geq 1$ on $\mathbb{D}$ and $f(0) = i$.

Which of the following are possible values

of $f\left(\frac{1}{2}\right)$?

- | | |
|---------|---------|
| 1. $-i$ | 2. i |
| 3. 1 | 4. -1 |

94. Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and let $f : \mathbb{D} \rightarrow \mathbb{D}$ be a holomorphic function. Suppose that $f(0) = 0$ and $f'(0) = 0$. Which of the following are possible

values of $f\left(\frac{1}{2}\right)$?

- | | |
|----------|-----------|
| 1. $1/4$ | 2. $-1/4$ |
| 3. $1/3$ | 4. $-1/3$ |

95. Let n be a positive integer. For a real number

$R > 1$, let $z(\theta) = Re^{i\theta}$, $0 \leq \theta < 2\pi$.

The set

$\{\theta \in [0, 2\pi) : |z(\theta)^n + 1| = |z(\theta)|^n - 1\}$ contains which of the following sets?

1. $\{\theta \in [0, 2\pi) : \cos n\theta = 1\}$
2. $\{\theta \in [0, 2\pi) : \sin n\theta = 1\}$
3. $\{\theta \in [0, 2\pi) : \cos n\theta = -1\}$
4. $\{\theta \in [0, 2\pi) : \sin n\theta = -1\}$

JUNE – 2021

PART – B

96. Let $f(z)$ be a non-constant entire function and $z = x + iy$. Let $u(x, y)$, $v(x, y)$ denote its real and imaginary parts respectively. Which of the following statements is FALSE?

1. $u_x = v_y$ and $u_y = -v_x$
2. $u_y = v_x$ and $u_x = -v_y$
3. $|f'(x + iy)|^2 = u_x(x, y)^2 + v_x(x, y)^2$
4. $|f'(x + iy)|^2 = u_y(x, y)^2 + v_y(x, y)^2$

97. Let f be a rational function of a complex variable z given by $f(z) = \frac{z^3 + 2z - 4}{z}$.

The radius of convergence of the Taylor series of f at $z = 1$ is

- | | |
|--------|-------------|
| 1. 0 | 2. 1 |
| 3. 2 | 4. ∞ |

98. Let γ be the positively oriented circle

$\{z \in \mathbb{C} : |z| = 3/2\}$.

Suppose that

$$\int_{\gamma} \frac{e^{iz}}{(z-1)(z-2i)^2} dz = 2\pi i C.$$

Then $|C|$ equals

- | | |
|----------|----------|
| 1. 2 | 2. 5 |
| 3. $1/2$ | 4. $1/5$ |

99. Let $\mathbb{D} \subset \mathbb{C}$ be the open disc $\{z \in \mathbb{C} : |z| < 1\}$ and $O(\mathbb{D})$ be the space of all holomorphic functions on $\mathbb{D}$. Consider the sets

$$A = \left\{ f \in O(\mathbb{D}) : f\left(\frac{1}{n}\right) = \begin{cases} e^{-n} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} ; \text{ for } n \geq 2 \right\}$$

$B = \{f \in O(\mathbb{D}) : f(1/n) = (n-2)/(n-1), n \geq 2\}$.

Which of the following statements is true?

1. Both A and B are non-empty
2. A is empty and B has exactly one element
3. A has exactly one element and B is empty
4. Both A, B are empty

PART – C

100. For any complex valued function f , let D_f denote the set on which the function f satisfies Cauchy-Riemann equations. Identify the functions for which D_f is equal to $\mathbb{C}$.

(1) $f(z) = \frac{z}{1+|z|}$

(2) $f(z) = (\cos \alpha x - \sin \alpha y) + i(\sin \alpha x + \cos \alpha y)$, where $z = x + iy$

(3) $f(z) = \begin{cases} e^{-\frac{1}{z^4}} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$

(4) $f(z) = x^2 + iy^2$, where $z = x + iy$

101. Let $\mathbb{T}$ denote the unit circle $\{z \in \mathbb{C} : |z| = 1\}$ in the complex plane and let $\mathbb{D}$ be the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$. Let R denote the set of points z_0 in $\mathbb{T}$ for which there exists a holomorphic function f in an open neighbourhood U_{z_0} of z_0 such that

$f(z) = \sum_{n=0}^{\infty} z^{4n}$ in $U_{z_0} \cap \mathbb{D}$. Then R contains

- (1) All points of $\mathbb{T}$
- (2) Infinitely many points of $\mathbb{T}$
- (3) All points of $\mathbb{T}$ except a finite set
- (4) No points of $\mathbb{T}$

102. Consider the function

$$f(z) = \frac{(\sin z)^m}{(1 - \cos z)^n} \text{ for } 0 < |z| < 1 \text{ where}$$

m, n are positive integers. Then $z = 0$ is

- (1) A removable singularity if $m \geq 2n$
- (2) A pole if $m < 2n$
- (3) A pole if $m \geq 2n$
- (4) An essential singularity for some values of m, n

103. Let f be an entire function such that

$$|zf(z) - 1 + e^z| \leq 1 + |z| \text{ for all } z \in \mathbb{C}. \text{ Then}$$

- (1) $f'(0) = -1$
- (2) $f'(0) = -1/2$
- (3) $f''(0) = -1/3$
- (4) $f''(0) = -1/4$

JUNE – 2022

PART – B

104. If $|e^{e^z}| = 1$ for a complex number $z = x + iy$, $x, y \in \mathbb{R}$, then which of the following is true?

1. $x = n\pi$ for some integer n
2. $y = (2n+1)\frac{\pi}{2}$ for some integer n
3. $y = n\pi$ for some integer n
4. $x = (2n+1)\frac{\pi}{2}$ for some integer n

105. Let $f(z) = (1-z)e^{\left(\frac{z^2}{2}\right)} = 1 + \sum_{n=1}^{\infty} a_n z^n$. Which of the following is false?

1. $f'(z) = -z^2 e^{\left(\frac{z^2}{2}\right)}$
2. $a_1 = a_2$
3. $a_n \in (-\infty, 0]$
4. $\sum_{n=3}^{\infty} |a_n| < 1$

106. Let f be a non-constant entire function such that $|f(z)| = 1$ for $|z| = 1$. Let U denote the open unit disk around 0. Which of the following is False?

1. $f(\mathbb{C}) = \mathbb{C}$
2. f has atleast one zero in U
3. f has atmost finitely many distinct zeroes in $\mathbb{C}$
4. f can have a zero outside U

107. For a positive integer n , let $f^{(n)}$ denote the n^{th} derivative of f . Suppose an entire function f satisfies $f^{(2)} + f = 0$. Which of the following is correct?

1. $(f^{(n)}(0))_{n \geq 1}$ is convergent
2. $\lim_{n \rightarrow \infty} f^{(n)}(0) = 1$
3. $\lim_{n \rightarrow \infty} f^{(n)}(0) = -1$
4. $(|f^{(n)}(0)|)_{n \geq 1}$ has a convergent subsequence

PART – C

108. For a bounded open connected subset Ω of $\mathbb{C}$, let $f : \Omega \rightarrow \mathbb{C}$ be holomorphic. Let (z_k) be a sequence of distinct complex numbers in Ω converging to z_0 . If $f(z_k) = 0$ for all $k \geq 1$ then which of the following statements are necessarily true?

1. If f is non-zero, then $z_0 \in \partial\Omega$
2. There exists $r > 0$ such that $f(z) = 0$ for every $z \in \Omega$ satisfying $|z - z_0| \leq r$
3. If $z_0 \in \Omega$, there exists $r > 0$ such that $f(z) = 0$ on $|z - z_0| = r$
4. $z_0 \in \partial\Omega$

109. Let f be an entire function such that $f(z)^2 + f'(z)^2 = 1$. Consider the following sets $X = \{z : f'(z) = 0\}$, $Y = \{z : f''(z) + f(z) = 0\}$. Which of the following statements are true?

1. Either X or Y has a limit point
2. If Y has a limit point, then f' is constant
3. If X has a limit point, then f is constant
4. $f(z) \in \{1, -1\}$ for all $z \in \mathbb{C}$

110. Let U be a bounded open set of $\mathbb{C}$ containing 0. Let $f : U \rightarrow U$ be holomorphic with $f(0) = 0$. For $n \in \mathbb{N}$, let f^n denote the composition of f done n times, that is, $f^n = \underbrace{f \circ \dots \circ f}_{n \text{ times}}$ while f' denotes the

derivative of f . Which of the following statements are true?

1. $(f^n)'(0) = (f'(0))^n$
2. $f^n(U) \subset U$
3. The sequence $((f'(0))^n)_{n \in \mathbb{N}}$ is bounded
4. $|f'(0)| \leq 1$

111. For an open subset Ω of $\mathbb{C}$ such that $0 \in \Omega$, which of the following statements are true?

1. $\{e^z : z \in \Omega\}$ is an open subset of $\mathbb{C}$
2. $\{|e^z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$
3. $\{\sin z : z \in \Omega\}$ is an open subset of $\mathbb{C}$
4. $\{|\sin z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$

JUNE – 2023

PART – B

112. Let C be the positively oriented circle in the complex plane of radius 3 centered at the origin. What is the value of the integral

$$\int_C \frac{dz}{z^2(e^z - e^{-z})}?$$

- (1) $i\pi/12$ (2) $-i\pi/12$
- (3) $i\pi/6$ (4) $-i\pi/6$

113. Consider the function f defined by

$$f(z) = \frac{1}{1 - z - z^2} \text{ for } z \in \mathbb{C} \text{ such that}$$

$1 - z - z^2 \neq 0$. Which of the following statements is true?

- (1) f is an entire function
- (2) f has a simple pole at $z = 0$
- (3) f has a Taylor series expansion

$f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0 = 1$, $a_1 = 0$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$

- (4) f has a Taylor series expansion

$f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0 = 1$, $a_1 = 1$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$

114. Let f be an entire function that satisfies $|f(z)| \leq e^y$ for all $z = x + iy \in \mathbb{C}$, where $x, y \in \mathbb{R}$. Which of the following statements is true?

- (1) $f(z) = ce^{-iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
- (2) $f(z) = ce^{iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
- (3) $f(z) = e^{-ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
- (4) $f(z) = e^{ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

115. Let $f(z) = \exp\left(z + \frac{1}{z}\right)$, $z \in \mathbb{C} \setminus \{0\}$. The residue of f at $z = 0$ is

- (1) $\sum_{l=0}^{\infty} \frac{1}{(l+1)!}$
- (2) $\sum_{l=0}^{\infty} \frac{1}{l!(l+1)}$
- (3) $\sum_{l=0}^{\infty} \frac{1}{l!(l+1)!}$
- (4) $\sum_{l=0}^{\infty} \frac{1}{(l^2 + l)!}$

PART – C

116. Let $f(z)$ be an entire function on $\mathbb{C}$. Which of the following statements are true?

- (1) $f(\bar{z})$ is an entire function
- (2) $\overline{f(z)}$ is an entire function
- (3) $\overline{f(\bar{z})}$ is an entire function
- (4) $\overline{f(z)} + f(\bar{z})$ is an entire function

117. Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and C the positively oriented boundary $\{|z| = 1\}$. Fix a finite set $\{z_1, z_2, \dots, z_n\} \subseteq \mathbb{D}$ of distinct points and consider the polynomial

$g(z) = (z - z_1)(z - z_2) \dots (z - z_n)$ of degree n . Let f be a holomorphic function in an open neighbourhood of $\mathbb{D}$ and define

$$P(z) = \frac{1}{2\pi i} \int_C f(\zeta) \frac{g(\zeta) - g(z)}{(\zeta - z)g(\zeta)} d\zeta.$$

Which of the following statements are true?

- (1) P is a polynomial of degree n
- (2) P is a polynomial of degree $n - 1$
- (3) P is a rational function on $\mathbb{C}$ with poles at $z_1, z_2, \dots, z_n$
- (4) $P(z_j) = f(z_j)$ for $j = 1, 2, \dots, n$.

118. Let $D = \{z \in \mathbb{C} : |z| < 1\}$. Consider the following statements.

- (a) $f : D \rightarrow D$ be a holomorphic function. Suppose α, β are distinct complex numbers in D such that $f(\alpha) = \alpha$ and $f(\beta) = \beta$. Then $f(z) = z$ for all $z \in D$.
- (b) There does not exist a bijective holomorphic function from D to the set of all complex numbers whose imaginary part is positive.

- (c) $f : D \rightarrow D$ be a holomorphic function. Suppose $\alpha \in D$ be such that $f(\alpha) = \alpha$ and $f'(\alpha) = 1$. Then $f(z) = z$ for all $z \in D$.

Which of the following options are true?

- (1) (a), (b) and (c) are all true.
 (2) (a) is true.
 (3) Both (a) and (b) are false.
 (4) Both (a) and (c) are true.
- 119.** Let $f : \{z : |z| < 1\} \rightarrow \{z : |z| \leq 1/2\}$ be a holomorphic function such that $f(0) = 0$. Which of the following statements are true?
- (1) $|f(z)| \leq |z|$ for all z in $\{z : |z| < 1\}$.
 (2) $|f(z)| \leq \frac{|z|}{2}$ for all z in $\{z : |z| < 1\}$
 (3) $|f(z)| \leq 1/2$ for all z in $\{z : |z| < 1\}$
 (4) It is possible that $f(1/2) = 1/2$.

DECEMBER – 2023

PART – B

- 120.** Let $\mathbb{H} = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$ denote the upper half plane and let $f : \mathbb{C} \rightarrow \mathbb{C}$ be defined by $f(z) = e^{iz}$. Which one of the following statements is true?
- (1) $f(\mathbb{H}) = \mathbb{C} \setminus \{0\}$.
 (2) $f(\mathbb{H}) \cap \mathbb{H}$ is countable.
 (3) $f(\mathbb{H})$ is bounded.
 (4) $f(\mathbb{H})$ is a convex subset of $\mathbb{C}$.
- 121.** How many roots does the polynomial $z^{100} - 50z^{30} + 40z^{10} + 6z + 1$ have in the open disc $\{z \in \mathbb{C} : |z| < 1\}$?
- (1) 100 (2) 50
 (3) 30 (4) 0
- 122.** Let f be a meromorphic function on an open set containing the unit circle C and its interior. Suppose that f has no zeros and no poles on C and let n_p and n_0 denote the number of poles and zeros of f inside C respectively. Which one of the following is true?
- (1) $\frac{1}{2\pi i} \int_C \frac{(zf)'}{zf} dz = n_0 - n_p + 1$.
 (2) $\frac{1}{2\pi i} \int_C \frac{(zf)'}{zf} dz = n_0 - n_p - 1$.
 (3) $\frac{1}{2\pi i} \int_C \frac{(zf)'}{zf} dz = n_0 - n_p$.

$$(4) \frac{1}{2\pi i} \int_C \frac{(zf)'}{zf} dz = n_p - n_0.$$

- 123.** Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be a real-differentiable function. Define $u, v : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $u(x, y) = \text{Re } f(x + iy)$ and $v(x, y) = \text{Im } f(x + iy)$, $x, y \in \mathbb{R}$.

Let $\nabla u = (u_x, u_y)$ denote the gradient. Which one of the following is necessarily true?

- (1) For $c_1, c_2 \in \mathbb{C}$, the level curves $u = c_1$ and $v = c_2$ are orthogonal wherever they intersect.
 (2) $\nabla u \cdot \nabla v = 0$ at every point.
 (3) If f is an entire function, then $\nabla u \cdot \nabla v = 0$ at every point.
 (4) If $\nabla u \cdot \nabla v = 0$ at every point, then f is an entire function.

PART – C

- 124.** Let $\Omega_1 = \{z \in \mathbb{C} : |z| < 1\}$ and $\Omega_2 = \mathbb{C}$. Which of the following statements are true?
- (1) There exists a holomorphic surjective map $f : \Omega_1 \rightarrow \Omega_2$.
 (2) There exists a holomorphic surjective map $f : \Omega_2 \rightarrow \Omega_1$.
 (3) There exists a holomorphic injective map $f : \Omega_1 \rightarrow \Omega_2$.
 (4) There exists a holomorphic injective map $f : \Omega_2 \rightarrow \Omega_1$.
- 125.** For every $n \geq 1$, consider the entire function $p_n(z) = \sum_{k=0}^n \frac{z^k}{k!}$. Which of the following statements are true?
- (1) The sequence of functions $(p_n)_{n \geq 1}$ converges to an entire function uniformly on compact subsets of $\mathbb{C}$.
 (2) For all $n \geq 1$, p_n has a zero in the set $\{z \in \mathbb{C} : |z| \leq 2023\}$.
 (3) There exists a sequence (z_n) of complex numbers such that $\lim_{n \rightarrow \infty} |z_n| = \infty$ and $p_n(z_n) = 0$ for all $n \geq 1$.
 (4) Let S_n denote the set of all the zeros of p_n . If $a_n = \min_{z \in S_n} |z|$, then $a_n \rightarrow \infty$ as $n \rightarrow \infty$.

126. Let X be an uncountable subset of $\mathbb{C}$ and let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an entire function. Assume that for every $z \in X$, there exists an integer $n \geq 1$ such that $f^{(n)}(z) = 0$. Which of the following statements are necessarily true?
- (1) $f = 0$.
 - (2) f is a constant function.
 - (3) There exists a compact subset K of $\mathbb{C}$ such that $f^{-1}(K)$ is not compact.
 - (4) f is a polynomial.

127. For an integer k , consider the contour integral $I_k = \int_{|z|=1} \frac{e^z}{z^k} dz$. Which of the following statements are true?
- (1) $I_k = 0$ for every integer k .
 - (2) $I_k \neq 0$ if $k \geq 1$.
 - (3) $|I_k| \leq |I_{k+1}|$ for every integer k .
 - (4) $\lim_{k \rightarrow \infty} |I_k| = \infty$.

JUNE – 2024**PART – B**

128. Consider the contour γ given by
- $$\gamma(\theta) = \begin{cases} e^{2i\theta} & \text{for } \theta \in [0, \pi/2] \\ 1 + 2e^{2i\theta} & \text{for } \theta \in [\pi/2, 3\pi/2] \\ e^{2i\theta} & \text{for } \theta \in [3\pi/2, 2\pi] \end{cases}$$
- Then what is the value of $\int_{\gamma} \frac{dz}{z(z-2)}$?
- (1) 0
 - (2) πi
 - (3) $-\pi i$
 - (4) $2\pi i$
129. Let f be an entire function. Which of the following statements is FALSE?
- (1) If $\operatorname{Re}(f)$, $\operatorname{Im}(f)$ are bounded then f is constant
 - (2) If $e^{|\operatorname{Re}(f)| + |\operatorname{Im}(f)|}$ is bounded, then f is constant
 - (3) If the sum $\operatorname{Re}(f) + \operatorname{Im}(f)$ and the product $\operatorname{Re}(f) \operatorname{Im}(f)$ are bounded, the f is constant
 - (4) If $\sin(\operatorname{Re}(f) + \operatorname{Im}(f))$ is bounded, then f is constant

130. Let a, b be two real numbers such that $a < 0 < b$. For a positive real number r , define $\gamma_r(t) = re^{it}$ (where $t \in [0, 2\pi]$) and
- $$I_r = \frac{1}{2\pi i} \int_{\gamma_r} \frac{z^2 + 1}{(z-a)(z-b)} dz.$$
- Which of

the following statements is necessarily true?

- (1) $I_r \neq 0$ if $r > \max\{|a|, b\}$
- (2) $I_r \neq 0$ if $r < \max\{|a|, b\}$
- (3) $I_r = 0$ if $r > \max\{|a|, b\}$ and $|a| = b$
- (4) $I_r = 0$ if $|a| < r < b$

131. For a complex number a such that $0 < |a| < 1$, which of the following statements is true?

- (1) If $|z| < 1$, then $|1 - \bar{a}z| < |z - a|$
- (2) If $|z - a| = |1 - \bar{a}z|$, then $|z| = 1$
- (3) If $|z| = 1$, then $|z - a| < |1 - \bar{a}z|$
- (4) If $|1 - \bar{a}z| < |z - a|$, then $|z| < 1$

PART – C

132. Which of the following conditions ensure that the power series $\sum_{n \geq 0} a_n z^n$ defines an entire function?

- (1) The power series converges for every $z \in \mathbb{C}$
- (2) The power series converges for every $z \in \mathbb{R}$
- (3) The power series converges for every $z \in \{2^n : n \in \mathbb{N}\}$
- (4) The power series converges for every $z \in \left\{ \frac{1}{5^n} : n \in \mathbb{N} \right\}$

133. Let f be an entire function such that for every integer $k \geq 1$ there is an infinite set X_k such that $f(z) = \frac{1}{k}$ for all $z \in X_k$. Which

of the following statements are necessarily true?

- (1) There exists an infinite set X such that $f(z) = 0$ for all $z \in X$
- (2) There exists a non-empty closed set X such that $f(z) = 0$ for all $z \in X$
- (3) The set X_k is unbounded for each $k \geq 1$
- (4) If there exists a bounded sequence $(z_k)_{k \geq 1}$ such that $z_k \in X_k$ for each $k \geq 1$, then f has a zero

134. Suppose that f is an entire function such that $|f(z)| \geq 2024$ for all $z \in \mathbb{C}$. Which of the following statements are necessarily true?

- (1) $f(z) = 2024$ for all $z \in \mathbb{C}$
- (2) f is a constant function
- (3) f is an injective function

(4) f is a bijective function

135. For $z \in \mathbb{C} \setminus \{0\}$, let $f(z) = \frac{1}{z} \sin\left(\frac{1}{z}\right)$ and $g(z) = f(z) \sin(z)$. Which of the following statements are true?
- (1) f has an essential singularity at 0
 - (2) g has an essential singularity at 0
 - (3) f has a removable singularity at 0
 - (4) g has a removable singularity at 0

DECEMBER – 2024

PART – B

136. Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be the function defined by $f(z) = e^{(\cos(1+i)) \sin z}$. For $z = x + iy \in \mathbb{C}$, write $f(z)$ as $u(x, y) + iv(x, y)$, where u, v are real-valued functions. Which of the following is the value of $\frac{\partial u}{\partial x}(0, 0)$?

- (1) 0
- (2) $\left(e + \frac{1}{e}\right) \frac{\cos 1}{2}$
- (3) $\left(e - \frac{1}{e}\right) \frac{\cos 1}{2}$
- (4) 1

137. Let $\mathbb{D} = \{z = x + iy \in \mathbb{C} : |z| < 1\}$ be the open unit disc and $f: \mathbb{D} \rightarrow \mathbb{C}$ a holomorphic function such that $f(0) = 0$. Let $\psi(z) = |f(z)|^2$, and $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$.

Which of the following statements is FALSE?

- (1) f can be extended to $\mathbb{C}$ as an entire function.
 - (2) f must have infinitely many zeros in $\mathbb{D}$
 - (3) f is not a polynomial.
 - (4) $\exp(f)$ cannot take every complex value.
138. For integers

$$m, n \geq 1, \text{ let } I_{m,n} = \frac{1}{2\pi i} \int_C z^m \bar{z}^n dz, \text{ where } C$$

is the circle $\{z \in \mathbb{C} : |z| = 1\}$ oriented counter clockwise. Which of the following statements is true?

- (1) $I_{m,n} = 1$ if $m = n$
- (2) $I_{m,n} = 1$ if $m + 1 = n$
- (3) $I_{m,n} = 1$ if $m = n + 1$
- (4) $I_{m,n} = 1$ if $m = n + 2$

139. Let $\mathbb{H} = \{z = x + iy \in \mathbb{C} : y > 0\}$ and $f: \mathbb{H} \rightarrow \mathbb{C}$ be a non-constant holomorphic function satisfying $|f(z)| < 1$ for all $z \in \mathbb{H}$. Which of the following statements is true?

- (1) $\lim_{y \rightarrow +\infty} f'(iy) = 0$
- (2) $\lim_{y \rightarrow +\infty} f'(iy)$ is a complex number with absolute value 1.
- (3) $\lim_{y \rightarrow +\infty} |f'(iy)| = +\infty$
- (4) $\lim_{y \rightarrow +\infty} f'(iy)$ is not a real number.

PART – C

140. Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be an entire function such that $f(z) = f(iz)$ for all $z \in \mathbb{C}$. Which of the following statements are true?

- (1) $f(z) = f(-z)$ for all $z \in \mathbb{C}$.
- (2) $f'(0) = f''(0) = f'''(0) = 0$
- (3) There is an entire function $g: \mathbb{C} \rightarrow \mathbb{C}$ such that $f(z) = g(z^4)$ for all $z \in \mathbb{C}$.
- (4) f is necessarily a constant function.

141. Let $p(z)$ be a non-constant polynomial over $\mathbb{C}$. Given $R > 0$, let $S_R = \{z \in \mathbb{C} : |P(z)| < R\}$. Which of the following statements are true?

- (1) S_R is an open subset of $\mathbb{C}$
- (2) S_R is a bounded subset of $\mathbb{C}$
- (3) $|P(z)| = R$ for every z on the boundary of S_R .
- (4) Every connected component of S_R contains a zero of $p(z)$.

142. Let disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and f be a holomorphic function on $\mathbb{D}$ such that the function $g(z) = e^{1/z} f(z)$ on $\mathbb{D} \setminus \{0\}$ is bounded. Which of the following statements are true?
- (1) $f(0) = 0$
 - (2) $f(z) = 0$ for all $z \in \mathbb{D}$
 - (3) There exists a nonzero constant c such that $f(z) = ce^{-1/z}$ for all $z \in \mathbb{D} \setminus \{0\}$.

- (4) There exists a nonzero constant c and a positive integer n such that

$$f(z) = cz^n e^{-1/z} \text{ for all } z \in \mathbb{D} \setminus \{0\}.$$

143. Let $f: \mathbb{C} \setminus \{-1, 1\} \rightarrow \mathbb{C}$ be a holomorphic function that does not take any value in the set $\{z \in \mathbb{C} : |z-1| < 1\}$. Which of the following statements are true?

- (1) f is constant.
- (2) f has removable singularities at -1 and 1 .
- (3) f is bounded.
- (4) f has either poles or essential singularities at -1 and 1 .

JUNE – 2025

PART – B

144. Let f be an entire function such that $f(\mathbb{C}) \subset \{x + iy \mid y = x + 1\}$. Which of the following statements is true?

- (1) $|f(z)| \rightarrow \infty$ as $|z| \rightarrow \infty$.
- (2) $\frac{f(z)}{z} \rightarrow 0$ as $|z| \rightarrow \infty$
- (3) $zf(z) \rightarrow 0$ as $|z| \rightarrow \infty$
- (4) $f(z) \rightarrow 0$ as $|z| \rightarrow \infty$

145. Which of the following statements is true?

- (1) There exists an entire function f such that $f^{(n)}(0) = \frac{n!}{n^n}$ for all positive integers n .
- (2) There exists an entire function f such that $f^{(n)}(0) = n!n^n$ for all positive integers n .
- (3) There exists an entire function f such that $f^{(n)}(0) = (n-1)!$ for all positive integers n .
- (4) There exists an entire function f such that $f^{(n)}(0) = n!n$ for all positive integers n .

146. Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be a polynomial map. For $R > 0$, let $\gamma_R: [0, 1] \rightarrow \mathbb{C}$ be the map $t \mapsto Re^{2\pi i t}$. Suppose that there exists $c \in \mathbb{R}$ such that

$$\int_0^1 |(f \circ \gamma_R)(t) \gamma_R'(t)| dt \rightarrow c \text{ as } R \rightarrow \infty.$$

Which of the following statements is False?

- (1) The function $zf(1/z) \rightarrow 0$ as $|z| \rightarrow \infty$.
- (2) The function f is constant.
- (3) $c = 0$
- (4) $c > 0$

147. Let X be the image of the interval $[0, 1]$ under the Möbius transformation

$$f(z) = \frac{z-i}{z+i}.$$

Which of the following statements is true?

- (1) X is the line segment joining -1 and $-i$.

(2) $X = \left\{ e^{i\theta} \mid \theta \in \left[\pi, \frac{3\pi}{2} \right] \right\}.$

- (3) X is the line segment joining -1 to 1 .

(4) $X = \left\{ e^{i\theta} \mid \theta \in \left[-\frac{\pi}{2}, \pi \right] \right\}.$

PART – C

148. Let $D^* = \{z \in \mathbb{C} : 0 < |z| < 1\}$ be the punctured unit disk and f be a bijective holomorphic map of D^* onto itself. Which of the following statements are true?

1. $\lim_{z \rightarrow 0} f(z)$ does not exist.
2. $\lim_{z \rightarrow 0} f(z)$ exists and has absolute value ≤ 1 .
3. $\lim_{z \rightarrow 0} f(z) = 0$
4. There exists $\theta \in \mathbb{R}$ such that $f(z) = e^{i\theta} z$ for all $z \in D^*$

149. Let f be an entire function. Which of the following are true?

1. If $f(z) = f(z+1)$ for all $z \in \mathbb{C}$ then f is a constant function.
2. If $f(z) = f(z+1) = f(z+i)$ for all $z \in \mathbb{C}$ then f is a constant function
3. If $f\left(\frac{1}{z}\right)$ has a removable singularity at 0 then f is a constant function
4. If f is a non-constant function then $f\left(\frac{1}{z}\right)$ has a pole at 0 .

150. Let f be an entire function which is not a polynomial. Let $A = \{\alpha \in \mathbb{C} \mid f^{(n)}(\alpha) \neq 0 \text{ for all } n \geq 0\}$. Which of the following statements are true?
1. A is nonempty
 2. A is finite
 3. A is infinite
 4. A is uncountable.

151. Let $\gamma : [0, 1] \rightarrow \mathbb{C}$ be the function $t \rightarrow e^{2\pi i t}$

and $I = \int_{\gamma} e^z e^{\frac{1}{z}} dz.$

Which of the following statements are true?

1. $I = 0$

2. $\frac{1}{2\pi i} I \in \{4n : n \in \mathbb{Z}, n \geq 1\}$

3. $I = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!}$

4. $I = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}$



ANSWERS

- | | | |
|---------------|----------------|----------------|
| 1. (2) | 2. (1) | 3. (1,4) |
| 4. (3) | 5. (2,3,4) | 6. (1,2) |
| 7. (1) | 8. (1) | 9. |
| 10. (2) | 11. (1,2,3) | 12. (1,3) |
| 13. (3) | 14. (3) | 15. (2) |
| 16. (1,2,4) | 17. (1,2,3,4) | 18. (1,2,3) |
| 19. (1) | 20. (3) | 21. (2) |
| 22. (1,2,3,4) | 23. (1,2,3,4) | 24. (2,3) |
| 25. (3) | 26. (2) | 27. (2) |
| 28. (3) | 29. (1,2,4) | 30. (3,4) |
| 31. (1,2,3,4) | 32. (1,2,4) | 33. (3) |
| 34. (3) | 35. (4) | 36. (2) |
| 37. (2,3) | 38. (2,4) | 39. (1,2) |
| 40. (1,2,4) | 41. (3) | 42. (2) |
| 43. (1) | 44. (3) | 45. (1,2) |
| 46. (3,4) | 47. (2,4) | 48. (2,4) |
| 49. (4) | 50. (3) | 51. (4) |
| 52. (3) | 53. (1,3) | 54. (1,2,3,4) |
| 55. (1,2,3) | 56. (1,3) | 57. (2) |
| 58. (1) | 59. (4) | 60. (3) |
| 61. (1,3) | 62. (1,2,3,4) | 63. (1,2,4) |
| 64. (1,3) | 65. (3) | 66. (1) |
| 67. (2) | 68. (4) | 69. (1,2) |
| 70. (1,3) | 71. (2,4) | 72. (1) |
| 73. (1) | 74. (2) | 75. (3) |
| 76. (1,4) | 77. (1,2,3,4) | 78. (2,4) |
| 79. (1,3) | 80. (4) | 81. (3) |
| 82. (3) | 83. (3) | 84. (1) |
| 85. (1,2,4) | 86. (1,2,3,4) | 87. (1,3,4) |
| 88. (2) | 89. (2) | 90. (1) |
| 91. (1) | 92. (4) | 93. (2) |
| 94. (1,2) | 95. (3) | 96. (2) |
| 97. (2) | 98. (4) | 99. (2) |
| 100. (3) | 101. (2,3) | 102. (1,2) |
| 103. (2,3) | 104. (2) | 105. (4) |
| 106. (4) | 107. (4) | 108. (1,3) |
| 109. (1,3) | 110. (1,2,3,4) | 111. (1,2,3) |
| 112. (4) | 113. (4) | 114. (1) |
| 115. (3) | 116. (3) | 117. (4) |
| 118. (2,4) | 119. (1,2,3) | 120. (3) |
| 121. (3) | 122. (1) | 123. (3) |
| 124. (1,3) | 125. (1,3,4) | 126. (4) |
| 127. (2) | 128. (3) | 129. (4) |
| 130. (3) | 131. (2) | 132. (1,2,3) |
| 133. (3,4) | 134. (2) | 135. (1,2) |
| 136. (2) | 137. (3) | 138. (2) |
| 139. (1) | 140. (1,2,3) | 141. (1,2,3,4) |
| 142. (1,2) | 143. (1,2,3) | 144. (2) |
| 145. (1) | 146. (4) | 147. (2) |
| 148. (2,3,4) | 149. (2,3) | 150. (1,3,4) |
| 151. (4) | | |