NUMERICAL ANALYSIS (PREVIOUS PAPERS NET)

JUNE - 2014

PART – B

1. Let f(x) = ax+b for $a, b \in \mathbb{R}$. Then the iteration $x_{n+1} = f(x_n)$ starting from any given x_0 for $n \geq 0$ converges

1. for all $a \in \mathbb{R}$
2. for no $a \in \mathbb{R}$
3. for $a \in [0,1)$
4. only for a=0

PART – C

2. Consider the function $f(x) = \sqrt{2+x}$ for $x \geq -2$ and the iteration $x_{n+1} = f(x_n);\ n \geq 0$ for x_0=1. What are the possible limits of the iteration?

1. $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$
2. -1
3. 2
4. 1

3. Consider the iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right),\ n \geq 0$ for a given $x_0 \neq 0$. Then

1. x_n converges to $\sqrt{2}$ with rate of convergence 1.
2. x_n converges to $\sqrt{2}$ with rate of convergence 2.
3. The given iteration is the fixed point iteration for f(x) = x^2-2.
4. The given iteration is the Newton's method for f(x) = x^2-2.

DECEMBER - 2014

PART - C

4. Let f :$\mathbb{R} \to \mathbb{R}$be a smooth function with non-vanishing derivative. The Newton's method for finding a root of f(x) =0 is the same as

1. fixed point iteration for the map g(x) = x- $f(x)/f'(x)$
2. Forward Euler method with unit step length for the differential equation $\frac{dy}{dx} + \frac{f(y)}{f'(y)} = 0$
3. fixed point iteration for g(x) = x + f(x)
4. fixed point iteration for g(x) = x - f(x)

5. Which of the following approximations for estimating the derivative of a smooth function f at a point x is of order 2 (i.e., the error term is $O(h^2)$)

1. $f'(x) \approx \frac{f(x+h) - f(x)}{h}$
2. $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$
3. $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$
4. $f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$

6. Let y(t) satisfy the differential equation $y' = \lambda y;\ y(0) = 1$. Then the backward Euler method, for $n \geq 1$ and h > 0 $\frac{y_n - y_{n-1}}{h} = \lambda y_n;\ y_0 = 1$ yields

1. a first order approximation to $e^{\lambda n h}$
2. a polynomial approximation to $e^{\lambda n h}$
3. a rational function approximation to $e^{\lambda n h}$
4. a Chebyshev polynomial approximation to $e^{\lambda n h}$

JUNE - 2015

PART - C

7. The following numerical integration formula is exact for all polynomials of degree less than or equal to 3

1. Trapezoidal rule
2. Simpson's $\frac{1}{3}rd$ rule
3. Simpson's $\frac{3}{8}th$ rule
4. Gauss- Legendre 2 point formula

DECEMBER - 2015

PART – B

8. Let f(x) = ax + 100 for a∈ℝ. Then the iteration $x_{n+1} = f(x_n)$ for n ≥ 0 and $x_0 = 0$ converges for
1. a = 5
2. a = 1
3. a = 0.1
4. a = 10

PART – C

9. The iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right),\ n \geq 0$ for a given $x_0 \neq 0$ is an instance of

1. fixed point iteration for f(x) = $x^2 - 2$
2. Newton's method for f(x) = $x^2 - 2$
3. fixed point iteration for $f(x) = \frac{x^2+2}{2x}$
4. Newton's method for f(x) = $x^2 + 2$

10. Let $f(x) = \sqrt{x+3}$ for x ≥ -3. Consider the iteration $x_{n+1} = f(x_n)$, $x_0 = 0$; n ≥ 0 The possible limits of the iteration are

1. -1
2. 3
3. 0
4. $\sqrt{3+\sqrt{3+\sqrt{3+...}}}$

JUNE - 2016

PART – B

11. Let $f(x) = x^2 + 2x + 1$ and the derivative of f at x= 1 is approximated by using the central-difference formula $f'(1) \approx \frac{f(1+h) - f(1-h)}{2h}$ with $h = \frac{1}{2}$ .Then the absolute value of the error in the approximation of $f'(1)$ is equal to
1. 1
2. 1/2
3. 0
4. 1/12

PART – C

12. Let H(x) be the cubic Hermite interpolation of f(x) = $x^4 + 1$ on the interval I = [0,1] interpolating at x = 0 and x = 1. Then

1. $\max_{x\in I} |f(x)-H(x)| = \frac{1}{16}.$

2. The maximum of $|f(x) – H(x)|$ is attained at $x = \frac{1}{2}$.

3. $\max_{x\in I} |f(x)-H(x)| = \frac{1}{21}.$

4. The maximum of $|f(x)–H(x)|$ is attained at $x = \frac{1}{4}$.

13. Consider the Runge-Kutta method of the form $y_{n+1} = y_n + ak_1 + bk_2$
$k_1 = hf(x_n, y_n)$
$k_2 = hf(x_n + \alpha h, y_n + \beta k_1)$ to approximate the solution of the initial value problem $y'(x) = f(x, y(x)), y(x_0) = y_0$. Which of the following choices of a, b, α and β yield a second order method?

1. $a=\frac{1}{2}, b=\frac{1}{2}, \alpha=1, \beta=1$
2. $a=1, b=1, \alpha=\frac{1}{2}, \beta=\frac{1}{2}$
3. $a=\frac{1}{4}, b=\frac{3}{4}, \alpha=\frac{2}{3}, \beta=\frac{2}{3}$
4. $a=\frac{3}{4}, b=\frac{1}{4}, \alpha=1, \beta=1$

14. Let $f : [0,3] \to \mathbb{R}$ be defined by $f(x) = |1 - |x – 2||$, where, $|\cdot|$ denotes the absolute value. Then for the numerical approximation of $\int_0^3 f(x)dx$, which of the following statements are true?

1. The composite trapezoid rule with three equal subintervals is exact.
2. The composite midpoint rule with three equal subintervals is exact.
3. The composite trapezoid rule with four equal subintervals is exact.
4. The composite midpoint rule with four equal subintervals is exact.

DECEMBER - 2016

PART - B

15. The values of α and β, such that $x_{n+1} = \alpha x_n\left(3-\frac{x_n^2}{a}\right)+\beta x_n\left(1+\frac{a}{x_n^2}\right)$ has 3rd order convergence to $\sqrt{a}$, are

1. $\alpha=\frac{3}{8}, \beta=\frac{1}{8}.$
2. $\alpha=\frac{1}{8}, \beta=\frac{3}{8}.$
3. $\alpha=\frac{2}{8}, \beta=\frac{2}{8}.$
4. $\alpha=\frac{1}{4}, \beta=\frac{3}{4}.$

PART - C

16. The order of linear multi step method $u_{j+1} = (1-a)u_j + au_{j-1} + \frac{h}{4}\{(a+3)u'_{j+1} + (3a+1)u'_{j-1}\}$ for solving $u' = f(x,u)$ is

1. 2 if a = -1
2. 2 if a = -2
3. 3 if a = -1
4. 3 if a = -2

JUNE - 2017

PART - B

17. The magnitude of the truncation error for the scheme $f'(x) = Af(x) + Bf(x+h) + Cf(x+2h)$ is equal to

1. $h^2 f'''(\xi)$ if $A = -\frac{5}{6h},\ B = \frac{3}{2h},\ C = -\frac{2}{3h}.$
2. $h^2 f'''(\xi)$ if $A = \frac{5}{6h},\ B = \frac{3}{2h},\ C = \frac{2}{3h}.$
3. $h^2 f''(x)$ if $A = -\frac{5}{6h},\ B = \frac{3}{2h},\ C = -\frac{2}{3h}.$
4. $h^2 f''(x)$ if $A = \frac{5}{6h},\ B = \frac{3}{2h},\ C = \frac{2}{3h}.$

DECEMBER – 2017

PART - B

18. The iterative method $x_{n+1} = g(x_n)$ for the solution of $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root $x = 2$ if g(x) equals

1. $x^2 - 2$ 2. $(x-2)^2 - 6$ 3. $1 + \frac{2}{x}$ 4. $\frac{x^2+2}{2x-1}$

PART - C

19. Consider the linear system Ax=b with $A = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 2 & -2 \\ -3 & -2 & 1 \end{bmatrix}$. Let x_n denote the nth Gauss-Seidel iteration and $e_n = x_n - x$. Let M be the corresponding matrix such that $e_{n+1} = Me_n$, $n \geq 0$. Which of the following statements are necessarily true?

1. all eigenvalues of M have absolute value less than 1
2. there is an eigenvalues of M with absolute value at least 1
3. e_n converges to 0 as $n \to \infty$ for all $b \in \mathbb{R}^3$ and any e_0
4. e_n does not converge to 0 as $n \to \infty$ for any $b \in \mathbb{R}^3$ unless $e_0 = 0$

20. For $f \in C[0,1]$ and $n > 1$, let $T(f) = \frac{1}{n}\left[\frac{1}{2}f(0) + \frac{1}{2}f(1) + \sum_{j=1}^{n-1} f\left(\frac{j}{n}\right)\right]$ be an approximation of the integral $I(f) = \int_0^1 f(x)dx.$ For which of the following functions f is $T(f) = I(f)$?

1. $1 + \sin 2\pi nx$ 2. $1 + \cos 2\pi nx$ 3. $\sin^2 2\pi nx$ 4. $\cos^2 2\pi(n+1)x$

JUNE - 2018

PART – B

21. The values of a,b,c such that $\int_0^h f(x)dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$ is exact for polynomials f of degree as high as possible are

1. $a = 0,\ b = \frac{3}{4},\ c = \frac{1}{4}$
2. $a = \frac{3}{4},\ b = \frac{2}{4},\ c = \frac{1}{4}$
3. $a = \frac{-2}{4},\ b = \frac{3}{4},\ c = \frac{1}{4}$
4. $a = 0,\ b = \frac{1}{4},\ c = \frac{3}{4}$

PART - C

22. Assume that a non-singular matrix A = L + D + U, where L and U are lower and upper triangular matrices respectively with all diagonal entries are zero, and D is a diagonal matrix. Let x* be the solution of Ax = b. Then the Gauss-Seidel iteration method $x^{(k+1)} = Hx^{(k)} + c$, k = 0, 1, 2, … with ||H|| < 1 converges to x* provided H is equal to

1. $-D^{-1}(L + U)$
2. $-(D + L)^{-1}U$
3. $-D(L + U)^{-1}$
4. $-(L - D)^{-1}U$

23. The forward difference operator is defined as $\Delta U_n = U_{n+1} - U_n$. Then which of the following difference equations has an unbounded general solution?

1. $\Delta^2 U_n - 3\Delta U_n + 2U_n = 0$
2. $\Delta^2 U_n + \Delta U_n + \frac{1}{4}U_n = 0$
3. $\Delta^2 U_n - 2\Delta U_n + 2U_n = 0$
4. $\Delta^2 U_{n+1} - \frac{1}{3}\Delta^2 U_n = 0$

DECEMBER – 2018

PART – B

24. Let f(x) be a polynomial of unknown degree taking the values

x	0	1	2	3
f(x)	2	7	13	16

All the fourth divided differences are -1/6. Then the coefficient of x^3 is

1. 1/3
2. -2/3
3. 16
4. -1

PART - C

25. Let f : [0, 1] → [0, 1] be twice continuously differentiable function with a unique fixed point $f(x_*) = x_*$. For a given $x_0 \in (0, 1)$ consider the iteration $x_{n+1} = f(x_n)$ for $n \geq 0$.
If $L = \max_{x\in[0,1]} |f'(x)|$, then which of the following are true?

1. If L < 1, then x_n converges to x_*.
2. x_n converges to x_* provided $L \geq 1$.
3. The error $e_n = x_n - x_*$ satisfies $|e_{n+1}| < L|e_n|$.
4. If $f'(x_*) = 0$, then $|e_{n+1}| < C|e_n|^2$ for some C > 0.

26. Let u(x) satisfy the boundary value problem $(BVP)\begin{cases} u'' + u' = 0, & x \in (0,1) \\ u(0) = 0 \\ u(1) = 1 \end{cases}$

Consider the finite difference approximation to (BVP)

$$(BVP)_h \begin{cases} \frac{U_{j+1}-2U_j+U_{j-1}}{h^2}+\frac{U_{j+1}-U_{j-1}}{2h}=0, j=1,\ldots,N-1 \\ U_0=0 \\ U_N=1 \end{cases}$$

Here U_j is an approximation to $u(x_j)$, where x_j = jh, j = 0, …,N is a partition of [0, 1] with h = 1/N for some positive integer N. Then which of the following are true?

1. There exists a solution to $(BVP)_h$ of the form $U_j = ar^j + b$ for some a, b $\in \mathbb{R}$ with r ≠ 1 and r satisfying $(2+h)r^2 - 4r + (2-h)=0$
2. $U_j = (r^j - 1) / (r^N - 1)$ where r satisfies $(2 + h) r^2 - 4r + (2 - h) = 0$ and r ≠ 1
3. u is monotonic in x
4. U_j is monotonic in j.

JUNE – 2019

PART – B

27. Consider solving the following system by Jacobi iteration scheme
x + 2my – 2mx = 1
nx + y + nz = 2
2mx + 2my + z=1, where m, n $\in \mathbb{Z}$. With any initial vector, the scheme converges provided m, n, satisfy

1. m + n = 3
2. m > n
3. m< n
4. m = n

PART - C

28. The values of a, b, c so that the truncation error in the formula
$\int_{-h}^{h} f(x)\,dx = ahf(-h)+bhf(0)+ahf(h)+ch^2 f'(-h)-ch^2 f'(h)$ is minimum, are

1. $a=\frac{7}{15}, b=\frac{16}{15}, c=\frac{1}{15}$
2. $a=\frac{7}{15}, b=\frac{16}{15}, c=\frac{-1}{15}$
3. $a=\frac{7}{15}, b=\frac{-16}{15}, c=\frac{1}{15}$
4. $a=\frac{7}{15}, b=\frac{-16}{15}, c=\frac{-1}{15}$

29. Consider the equation $x^2 + ax + b = 0$ which has two real roots α and β. Then which of the following iteration scheme converges when x_0 is chosen sufficiently close to α?

1. $x_{n+1} = -\frac{ax_n+b}{x_n}, if\ |\alpha|>|\beta|$
2. $x_{n+1} = -\frac{x_n^2+b}{a}, if\ |\alpha|>1$
3. $x_{n+1} = -\frac{b}{x_n+a}, if\ |\alpha|<|\beta|$
4. $x_{n+1} = -\frac{x_n^2+b}{a}, if\ 2|\alpha|<|\alpha+\beta|$

DECEMBER – 2019

PART – B

30. Let x = ξ be a solution of $x^4 - 3x^2 + x - 10 = 0$. The rate of convergence for the iterative method $x_{n+1} = 10 - x_n^4 + 3x_n^2$ is equal to

1. 1
2. 2
3. 3
4. 4

PART - C

31. Consider the ordinary differential equation (ODE)

$$\begin{cases} y'(x)+y(x)=0, \quad x>0, \\ \quad y(0)=1. \end{cases}$$

and the following numerical scheme to solve the ODE

$$\begin{cases} \dfrac{Y_{n+1}-Y_{n-1}}{2h}+Y_{n-1}=0,\ n\geq 1, \\ Y_0=1, Y_1=1. \end{cases}$$

If $0<h<\frac{1}{2}$, then which of the following statements are true?

1. $(Y_n)\to\infty$ as $n\to\infty$
2. $(Y_n)\to 0$ as $n\to\infty$
3. (Y_n) is bounded
4. $\max_{nh\in[0,T]} |y(nh)-Y_n| \to \infty \ as\, T\to\infty$

32. The values of α, A, B, C for which the quadrature formula

$$\int_{-1}^{1}(1-x)\, f(x)\,dx = Af(-\alpha)+Bf(0)+Cf(\alpha)$$

is exact for polynomials of highest possible degree, are

1. $\alpha=\sqrt{\frac{3}{5}}, A=\frac{5}{9}+\frac{\sqrt{5}}{3\sqrt{3}}, B=\frac{8}{9}, C=\frac{5}{9}-\frac{\sqrt{5}}{3\sqrt{3}}$
2. $\alpha=\sqrt{\frac{3}{5}}, A=\frac{5}{9}-\frac{\sqrt{5}}{3\sqrt{3}}, B=\frac{8}{9}, C=\frac{5}{9}+\frac{\sqrt{5}}{3\sqrt{3}}$
3. $\alpha=\sqrt{\frac{3}{5}}, A=\frac{5}{9}\left(1-\frac{\sqrt{3}}{\sqrt{5}}\right), B=\frac{8}{9}, C=\frac{5}{9}\left(1+\frac{\sqrt{3}}{\sqrt{5}}\right)$
4. $\alpha=\sqrt{\frac{3}{5}}, A=\frac{5}{9}\left(1+\frac{\sqrt{3}}{\sqrt{5}}\right), B=\frac{8}{9}, C=\frac{5}{9}\left(1-\frac{\sqrt{3}}{\sqrt{5}}\right)$

DECEMBER – 2019 (Assam)

PART - B

33. Assume that a, b $\in\mathbb{R}\backslash\{0\}$ and $a^2\neq b^2$. Suppose that the Gauss-Seidel method is used to solve the system of equations $\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Then the set of all values of (a, b) such that the method converges for every choice of initial vector is

1. $\{(a, b) \mid a^2 < b^2\}$
2. $\{(a, b) \mid a < |b|\}$
3. $\{(a, b) \mid |b| < |a|\}$
4. $\{(a, b) \mid a^2 + b^2 < 1\}$

PART - C

34. Consider the first order initial value problem y′(x) = -y(x), x > 0, y(0) = 1 and the corresponding numerical scheme $4\left(\frac{y_{n+1}-y_{n-1}}{2h}\right)-3\left(\frac{y_{n+1}-y_n}{h}\right)=-y_n$, with y_0 = 1, $y_1 = e^{-h}$, where h is the step size. Then which of the following statements are true?

1. The order of the scheme is 1
2. The order of the scheme is 2
3. $|y_n - y(nh)| \to \infty$ as $n \to \infty$
4. $|y_n - y(nh)| \to 0$ as $n \to \infty$

35. Consider the integration formula

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}[f(x_0) + f(x_1)] + ph^2(f'(x_0) - f'(x_1)),$$

where $h = x_1 - x_0$. Then the constant p such that the above formula gives the exact value of the highest degree polynomial and the degree d of the corresponding polynomial are given by

1. $p = \frac{1}{6}, d = 4$
2. $p = \frac{1}{12}, d = 3$
3. $p = \frac{1}{6}, d = 3$
4. $p = \frac{1}{12}, d = 4$

JUNE – 2020

PART - B

36. Let f be an infinitely differentiable real-valued function on a bounded interval I. Take $n \geq 1$ interpolation points $\{x_0, x_1, \ldots, x_{n-1}\}$. Take n additional interpolation points
$x_{n+j} = x_j + \varepsilon$, $j = 0, 1, \ldots, n-1$
where $\varepsilon > 0$ is such that $\{x_0, x_1, \ldots, x_{2n-1}\}$ are all distinct.
Let p_{2n-1} be the Lagrange interpolation polynomial of degree $2n - 1$ with the interpolation points $\{x_0, x_1, \ldots, x_{2n-1}\}$ for the function f.
Let q_{2n-1} be the Hermite interpolation polynomial of degree $2n - 1$ with the interpolation points $\{x_0, x_1, \ldots, x_{n-1}\}$ for the function f. In the $\varepsilon \to 0$ limit, the quantity
$$\sup_{x \in I} |p_{2n-1}(x) - q_{2n-1}(x)|$$

1. does not necessarily converge
2. converges to $\frac{1}{2n}$
3. converges to 0
4. converges to $\frac{1}{2n+1}$

PART – C

37. Fix a $\alpha \in (0, 1)$. Consider the iteration defined by

(*) $x_{k+1} = \frac{1}{2}(x_k^2 + \alpha)$, k = 0, 1, 2, …

The above iteration has two distinct fixed points ζ_1 and ζ_2 such that $0 < \zeta_1 < 1 < \zeta_2$.
Which of the following statements are true?

1. The iteration (*) is equivalent to the recurrence relation $x_{k+2} - \zeta_1 = \frac{1}{2}(x_k + \zeta_1)(x_k - \zeta_1)$, k = 0, 1, 2, …
2. The iteration (*) is equivalent to the recurrence relation $x_{k+1} - \zeta_1 = \frac{1}{2}(x_k + \zeta_2)(x_k - \zeta_1)$, k = 0, 1, 2, …
3. If $0 \leq x_0 < \zeta_2$ then $\lim_{k \to \infty} x_k = \zeta_1$

4. If $-\zeta_2 < x_0 \le 0$ then $\lim_{k\to\infty} x_k = \zeta_1$

38. Consider the function $f : [0, 1] \to \mathbb{R}$ defined by

$$f(x) := \begin{cases} 2^{-\left\{1+\left(\log_2\left(\frac{1}{x}\right)\right)^{\frac{1}{\beta}}\right\}^{\beta}} & for\ x \in (0,1] \\ 0 & for\ x = 0, \end{cases}$$

where $\beta \in (0, \infty)$ is a parameter. Consider the iterations
$x_{k+1} = f(x_k)$, k = 0, 1, … ; $x_0 > 0$.
Which of the following statements are true about the iteration?
1. For $\beta = 1$, the sequence $\{x_k\}$ converges to 0 linearly with asymptotic rate of convergence $\log_{10}2$
2. For $\beta > 1$, the sequence $\{x_k\}$ does not converge to 0
3. For $\beta \in (0, 1)$, the sequence $\{x_k\}$ converges to 0 sublinearly
4. For $\beta \in (0, 1)$, the sequence $\{x_k\}$ converges to 0 superlinearly

JUNE – 2020 (Tamil Nadu)

PART - B

39. Consider the Newton-Raphson method applied to approximate the square root of a positive number α. A recursion relation for the error $e_n = x_n - \sqrt{\alpha}$ is given by

1. $e_{n+1} = \frac{1}{2}\left(e_n + \frac{\alpha}{e_n}\right)$
2. $e_{n+1} = \frac{1}{2}\left(e_n - \frac{\alpha}{e_n}\right)$
3. $e_{n+1} = \frac{1}{2}\frac{e_n^2}{e_n + \sqrt{\alpha}}$
4. $e_{n+1} = \frac{e_n^2}{e_n + 2\sqrt{\alpha}}$

PART – C

40. Consider the numerical integration formula
$\int_{-1}^{1} g(x)\,dx \approx g(\alpha) + g(-\alpha),$ where $\alpha = (0.2)^{1/4}$. Which of the following statements are true?

1. The integration formula is exact for polynomialss of the form a + bx, for all a, b $\in \mathbb{R}$
2. The integration formula is exact for polynomials of the form $a + bx + cx^2$, for all a, b, c $\in \mathbb{R}$
3. The integration formula is exact for polynomials of the form $a + bx + cx^2 + dx^3$, for all a, b, c, d $\in \mathbb{R}$
4. The integration formula is exact for polynomials of the form $a + bx + cx^3 + dx^4$ for all a, b, c, d $\in \mathbb{R}$

JUNE – 2021

PART – B

41. Let the solution to the initial value problem
$y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$
be computed using the Euler's method with step-length h = 0.4. If y(0.8) and w(0.8) denote the exact and approximate solutions at t = 0.8, then an error bound for Euler's method is given by
1. $0.2(0.5e^2 - 2)(e^{0.4} - 1)$
2. $0.1(e^{0.4} - 1)$
3. $0.2(0.5e^2 - 2)(e^{0.8} - 1)$

4. $0.1(e^{0.8} - 1)$

42. Let a, b, c $\in \mathbb{R}$ be such that the quadrature rule

$$\int_{-1}^{1} f(x)\,dx = af(-1) + bf'(0) + cf'(1)$$

is exact for all polynomials of degree less than or equal to 2. The a + b + c equal to

1. 4 2. 3 3. 2 4. 1

PART – C

43. The values of a, b, c, d, e for which the function

$$f(x) = \begin{cases} a(x-1)^2 + b(x-2)^3 & -\infty < x \le 2 \\ c(x-1)^2 + d & 2 \le x \le 3 \\ (x-1)^2 + e(x-3)^3 & 3 \le x < \infty \end{cases}$$

is a cubic spline are

1. a = c = 1, d = 0, b, e are arbitrary
2. a = b = c = 1, d = 0, e is arbitrary
3. a = b = c = d = 1, e is arbitrary
4. a = b = c = d = e = 1

44. Consider the Euler method for integration of the system of differential equations

$$\dot{x} = -y$$

$$\dot{y} = x$$

Assume that (x_i^n, y_i^n) are the points obtained for i = 0, 1, …, n^2 using a time-step h = 1/n starting at the initial point $(x_0, y_0) = (1, 0)$. Which of the following statements are true?

1. The points (x_i^n, y_i^n) lie on a circle of radius 1
2. $\lim_{n\to\infty}(x_n^n, y_n^n) = (\cos(1), \sin(1))$
3. $\lim_{n\to\infty}(x_2^n, y_2^n) = (1, 0)$
4. $(x_i^n)^2 + (y_i^n)^2 > 1$ for i ≥ 1

JUNE – 2022

PART - B

45. Let A be following invertible matrix with real positive entries $A = \begin{pmatrix} 1 & 2 \\ 8 & 9 \end{pmatrix}$. Let G be the associated Gauss-Seidel iteration matrix. What are the two eigenvalues of G?

1. 0 and $\frac{4}{3}$ 2. 0 and $-\frac{4}{3}$ 3.0 and $\frac{16}{9}$ 4. $\frac{4}{3}$ and $-\frac{4}{3}$

PART – C

46. Consider the ODE $\dot{x} = f(t, x)$ in $\mathbb{R}$, for a smooth function f. Consider a general second order Runge-Kutta formula of the form $x(t + h) = x(t) + w_1hf(t, x) + w_2hf(t + \alpha h, x + \beta hf) + 0(h^3)$. Which of the following choices of $(w_1, w_2, \alpha, \beta)$ are correct?

1. $\left(\frac{1}{2},\frac{1}{2},1,1\right)$ 2. $\left(\frac{1}{2},1,\frac{1}{2},1\right)$ 3. $\left(\frac{1}{4},\frac{3}{4},\frac{2}{3},\frac{2}{3}\right)$ 4. (0, 1, 1, 1)

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PART – B

47. Which of the following values of a, b, c and d will produce a quadrature formula

$$\int_{-1}^{1} f(x)\,dx \approx af(-1)+bf(1)+cf'(-1)+df'(1)$$

that has degree of precision 3?

1. $a=1, b=1, c=\frac{1}{3}, d=-\frac{1}{3}$
2. $a=-1, b=1, c=\frac{1}{3}, d=-\frac{1}{3}$
3. $a=1, b=1, c=-\frac{1}{3}, d=-\frac{1}{3}$
4. $a=1, b=-1, c=\frac{1}{3}, d=-\frac{1}{3}$

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PART – B

48. Using Euler's method with the step size 0.05, the approximate value of the solution for the initial value problem $\frac{dy}{dx}=\sqrt{3x+2y+1}, y(1)=1,$ at x = 1.1 (rounded off to two decimal places), is

1. 1.50 2. 1.65 3. 1.25 4. 1.15

PART – C

49. The coefficient of x^3 in the interpolating polynomial for the data

x	0	1	2	3	4
y	1	2	1	3	5

is

1. $-\frac{1}{3}$
2. $-\frac{1}{2}$
3. $\frac{5}{6}$
4. $\frac{17}{6}$

50. Consider the initial value problem

$$\frac{dy}{dx}=f(x,y), y(x_0)=y_0,$$

where f is a twice continuously differentiable function on a rectangle containing the point (x_0, y_0). With the step-size h, let the first iterate of a second order scheme to approximate the solution of the above initial value problem be given by $y_1 = y_0 + Pk_1 + Qk_2$,

where $k_1 = hf(x_0, y_0)$, $k_2 = hf(x_0 + \alpha_0 h, y_0 + \beta_0 k_1)$ and P, Q, $\alpha_0, \beta_0 \in \mathbb{R}$.
Which of the following statements are correct?

1. If $\alpha_0 = 2$, then $\beta_0 = 2, P=\frac{3}{4}, Q=\frac{1}{4}$
2. If $\beta_0 = 3$, then $\alpha_0 = 3, P=\frac{5}{6}, Q=\frac{1}{6}$
3. If $\alpha_0 = 2$, then $\beta_0 = 2, P=\frac{1}{4}, Q=\frac{3}{4}$
4. If $\beta_0 = 3$, then $\alpha_0 = 3, P=\frac{1}{6}, Q=\frac{5}{6}$

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PART – B

51. If the value of the approximate solution of the initial value problem

$$\begin{cases} y'(x) = x(y(x)+1), & x \in \mathbb{R} \\ y(0) = \beta \end{cases}$$

at x = 0.2 using the forward Euler method with step size 0.1 is 1.02, then the value of β is

1. 0 2. -1 3. 2 4. 1

PART – C

52. Let g(x) be the polynomial of degree at most 4 that interpolates the data

x	-1	0	2	3	6
y	-30	1	c	10	19

If g(4) = 5, then which of the following statements are true?

1. c = 13 2. g(5) = 6 3. g(1) = 14 4. c = 15

53. Let S denote the set of all 2 × 2 matrices A such that the iterative sequence generated by the Gauss-Seidel method applied to the system of linear equations $A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ converges for every initial guess. Then which of the following statements are true?

1. $\begin{pmatrix} 5 & 8 \\ 1 & 2 \end{pmatrix} \in S$ 2. $\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \in S$

3. $\begin{pmatrix} -3 & 1 \\ 2 & 3 \end{pmatrix} \in S$ 4. $\begin{pmatrix} 2 & 2 \\ 4 & 3 \end{pmatrix} \in S$

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PART – B

54. Let f: $\mathbb{R} \to \mathbb{R}$ be such that $\sup_{x \neq y} \frac{|f(x) - f(y)|}{|x-y|} = L,\ where 1 < L < \infty.\ Let\, h: \mathbb{R} \to \mathbb{R}$ be a differentiable function satisfying $|h'(x)| \leq \frac{3}{4}\ for\ all\ x \in \mathbb{R}$. For $\alpha > 0$, define $g(x) = \alpha f(x) + h(x)\ for\ x \in \mathbb{R}$. Consider the sequence $\{x_k\}_{k=0}^{\infty}$ defined by $x_{k+1} = g(x_k),\ k = 0,1,........,$ where $x_0 \in \mathbb{R}$. The sequence $\{x_k\}_{k=0}^{\infty}$ converges to the solution of the equation x=g(x) if

(1) $\alpha < \frac{2}{3L}$ (2) $\alpha < \frac{3}{2L}$ (3) $\alpha < 4L$ (4) $\alpha < \frac{1}{4L}$

PART – C

55. Let f(x) be the polynomial of degree at most 2 that interpolates the data (-1,2), (0,1) and (1,2). If g(x) is a polynomial of degree at most 3 such that f(x)+g(x) interpolates the data (-1,2), (0,1), (1,2) and (2,17), then

(1) f(5)+g(3)=50 (2) 2f(5)-g(3)=4

(3) f(1)+g(3)=50 (4) f(5)+g(3)=74

56. If $\lambda \in \mathbb{R}$ and $p \in \mathbb{R}$ are such that the quadrature formula

$$\int_{x_0}^{x_0+h} f(x)dx \approx \lambda h(f(x_0)+f(x_0+h)) + ph^3(f''(x_0)+f''(x_0+h))$$ is exact for all polynomials of degree as high as possible, then

(1) $2\lambda + 24p = 0$ (2) $7\lambda - 12p = 4$

(3) $2\lambda + 24p = -3$ (4) $7\lambda - 12p = 11$

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PART – B

57. If the function s : [0, 4] $\to\mathbb{R}$ defined by

$$s(x) = \begin{cases} a(x-2)^2 + b(x-1)^2, & 0 \le x \le 1, \\ (x-2)^2, & 1 < x \le 3, \\ 2c(x-2)^2 + (x-3)^3, & 3 < x \le 4 \end{cases}$$

is a cubic spline, then the value of 2a + b + 2c is

(1) 2 (2) 3 (3) 4 (4) 5

PART-C

58. If $\alpha, \beta \in \mathbb{R}$ are such that the equation

$$\int_0^3 f(x)dx = \frac{3}{2}[f(\alpha) + f(\alpha+\beta)]$$

holds for all polynomials f(x) of degree less than or equal to 2, then which of the following statements are true?

1. $(\alpha,\beta) = \left(\frac{3-\sqrt{3}}{2}, \sqrt{3}\right)$ or $(\alpha,\beta) = \left(\frac{3+\sqrt{3}}{2}, -\sqrt{3}\right)$

2. $(\alpha,\beta) = \left(\frac{3-\sqrt{2}}{2}, \sqrt{2}\right)$ or $(\alpha,\beta) = \left(\frac{3+\sqrt{2}}{2}, -\sqrt{2}\right)$

3. $(\alpha,\beta) = \left(\frac{3-\sqrt{5}}{2}, \sqrt{5}\right)$ or $(\alpha,\beta) = \left(\frac{3+\sqrt{5}}{2}, -\sqrt{5}\right)$

4. $(\alpha,\beta) = \left(\frac{3-\sqrt{7}}{2}, \sqrt{7}\right)$ or $(\alpha,\beta) = \left(\frac{3+\sqrt{7}}{2}, -\sqrt{7}\right)$

ANSWER KEY

1. (3)	2. (1,3)	3. (2,4)	4. (1,2)	5.(2,3,4)	6. (1, 3)
7. (2,3,4)	8. (3)	9. (2, 3)	10.(4)	11. (3)	12. (1,2)
13.(1,3)	14. (1,2)	15. (2)	16. (2,3)	17. (*)	18.(4)
19. (2,4)	20. (1)	21.(1)	22.(2)	23. (1,3,4)	24. (1)
25. (1,3,4)	26. (1,2,3,4)	27. (4)	28. (1)	29. (1,3,4)	30. (1)
31. (2,3)	32. (1,4)	33. (3)	34. (1,3)	35. (2)	36. (3)
37. (1)	38. (1,3)	39. (3)	40. (1)	41. (3)	42. (1)
43. (1,2)	44. (2,3,4)	45. (3)	46. (1,3)	47. (1)	48. (3)
49. (4)	50. (1,2)	51. (4)	52. (2,3,4)	53. (1,2,3)	54.(4)
55. (2,3,4)	56. (1,2)	57. (2)	58. (1)		