REAL ANALYSIS (PREVIOUS PAPERS NET)

JUNE – 2014

PART - B

1. Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ be given by f(x) = x^2. Then f is uniformly continuous if

1. A is bounded subset of $\mathbb{R}$.
2. A is a dense subset of $\mathbb{R}$.
3. A is an unbounded and connected subset of $\mathbb{R}$.
4. A is an unbounded and open subset of $\mathbb{R}$.

2. Let α, p be real numbers and $\alpha > 1$.

1. If p>1, then $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx < \infty$.
2. If $p > \frac{1}{\alpha}$, then $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx < \infty$.
3. If $p < \frac{1}{\alpha}$, then $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx < \infty$.
4. For any $p \in \mathbb{R}$, we have $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx = \infty$.

3. Let $f : X \to Y$ be a function from a metric space X to another metric space Y. For any Cauchy sequence $\{x_n\}$ in X,

1. if f is continuous then $\{f(x_n)\}$ is a Cauchy sequence in Y.
2. if $\{f(x_n)\}$ is Cauchy then $\{f(x_n)\}$ is always convergent in Y.
3. if $\{f(x_n)\}$ is Cauchy in Y then f is continuous.
4. $\{x_n\}$ is always convergent in X.

4. $\lim_{n\to\infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \ldots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} \right)$ equals

1. $\sqrt{2}$
2. $\frac{1}{\sqrt{2}}$
3. $\sqrt{2}$ +1
4. $\frac{1}{\sqrt{2}+1}$

5. Consider the following sets of functions on $\mathbb{R}$

W=The set of constant functions on $\mathbb{R}$

X=The set of polynomial functions on $\mathbb{R}$

Y=The set of continuous functions on $\mathbb{R}$

Z=The set of all functions on $\mathbb{R}$.

Which of these sets has the same cardinality as that of $\mathbb{R}$?

1. Only W.
2. Only W and X.
3. Only W,X and Z.
4. All of W,X,Y and Z.

6. Let p(x) be a polynomial in the real variable x of degree 5. Then $\lim_{n\to\infty} \frac{p(n)}{2^n}$ is

1. 5
2. 1
3. 0
4. ∞

7. For a continuous function $f : \mathbb{R} \to \mathbb{R}$ let $Z(f) = \{x \in \mathbb{R} : f(x) = 0\}$. Then Z(f) is always

1. compact
2. open
3. connected
4. Closed

PART – C

8. Let $X=\{(x,y)\in\mathbb{R}^2: x^2+y^2<5\}$ and $K=\{(x,y)\in\mathbb{R}^2 : 1\le x^2+y^2\le 2$ or $3\le x^2+y^2\le 4\}$. Then,
1. $X\setminus K$ has three connected components.
2. $X\setminus K$ has no relatively compact connected component in X.
3. $X\setminus K$ has two relatively compact connected components in X.
4. All connected components of $X\setminus K$ are relatively compact in X.

9. For two subsets X and Y of ℝ, let $X+Y=\{x+y: x\in X, y\in Y\}$.
1. If X and Y are open sets then X+Y is open.
2. If X and Y are closed sets then X+Y is closed.
3. If X and Y are compact sets then X+Y is compact.
4. If X is closed and Y is compact then X+Y is closed.

10. Let $\{f_n\}$ be a sequence of continuous functions on ℝ.
1. If $\{f_n\}$ converges to f pointwise on ℝ, then $\lim_{n\to\infty}\int_{-\infty}^{\infty} f_n(x)dx=\int_{-\infty}^{\infty} f(x)dx$.
2. If $\{f_n\}$ converges to f uniformly on ℝ then $\lim_{n\to\infty}\int_{-\infty}^{\infty} f_n(x)dx=\int_{-\infty}^{\infty} f(x)dx$.
3. If $\{f_n\}$ converges to f uniformly on ℝ, then f is continuous on ℝ.
4. There exists a sequence of continuous functions $\{f_n\}$ on ℝ, such that $\{f_n\}$ converges to f uniformly on ℝ, but $\lim_{n\to\infty}\int_{-\infty}^{\infty} f_n(x)dx\neq\int_{-\infty}^{\infty} f(x)dx$.

11. Let $\{a_n\}$, $\{b_n\}$ be given bounded sequences of positive real numbers. Then (Here $a_n\uparrow a$ means a_n increase to a as n goes to ∞, similarly, $b_n\downarrow b$ means b_n decreases to b as n goes to ∞
1. if $a_n\uparrow a$, then $\sup_{n\ge1}(a_nb_n)=a(\sup_{n\ge1} b_n)$.
2. if $a_n\uparrow a$, then $\sup_{n\ge1}(a_nb_n)<a(\sup_{n\ge1} b_n)$.
3. if $b_n\downarrow b$, then $\inf_{n\ge1}(a_nb_n)=(\inf_{n\ge1} a_n)b$.
4. if $b_n\downarrow b$, then $\inf_{n\ge1}(a_nb_n)>(\inf_{n\ge1} a_n)b$.

12. Let $S\subset\mathbb{R}^2$ be defined by $S=\left\{\left(m+\frac{1}{2^{|p|}}, n+\frac{1}{2^{|q|}}\right): m,n,p,q\in\mathbb{Z}\right\}$. Then,
1. S is discrete in $\mathbb{R}^2$.
2. the set of limit points of S is the set $\{(m,n): m,n\in\mathbb{Z}\}$.
3. $\mathbb{R}^2\setminus S$ is connected but not path connected.
4. $\mathbb{R}^2\setminus S$ is path connected.

13. Let a,b,c be positive real numbers, $D=\{(x_1,x_2,x_3)\in\mathbb{R}^3: x_1^2+x_2^2+x_3^2\le 1\}$,

$E=\{(x_1,x_2,x_3)\in\mathbb{R}^3: \frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}+\frac{x_3^2}{c^2}\le 1\}$ and $A=\begin{bmatrix} a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$, det A > 1. Then, for a compactly supported continuous function f on $\mathbb{R}^3$, which of the following are correct?

1. $\int_D f(Ax)dx = \int_E f(x)dx$
2. $\int_D f(Ax)dx = \frac{1}{abc}\int_D f(x)dx$
3. $\int_D f(Ax)dx = \frac{1}{abc}\int_E f(x)dx$
4. $\int_{\mathbb{R}^3} f(Ax)\, dx = \frac{1}{abc}\int_{\mathbb{R}^3} f(x)\, dx$

14. Let f: (0,1)→ℝ be continuous. Suppose that |f(x) – f(y)|≤|sinx – siny| for all $x, y \in (0,1)$. Then

1. f is discontinuous at least at one point in (0,1).
2. f is continuous everywhere on (0,1), but not uniformly continuous on (0,1).
3. f is uniformly continuous on (0,1).
4. $\lim_{x\to 0^+} f(x)$ exists.

15. Let $p_n(x) = a_n x^2 + b_n x + c_n$ be a sequence of quadratic polynomials where $a_n, b_n, c_n \in \mathbb{R}$, for all n ≥ 1. Let $\lambda_0, \lambda_1, \lambda_2$ be distinct real numbers such that $\lim_{n\to\infty} p_n(\lambda_0) = A_0$, $\lim_{n\to\infty} p_n(\lambda_1) = A_1$ and $\lim_{n\to\infty} p_n(\lambda_2) = A_2$. Then

1. $\lim_{n\to\infty} p_n(x)$ exists for all $x \in \mathbb{R}$.
2. $\lim_{n\to\infty} p_n'(x)$ exists for all $x \in \mathbb{R}$.
3. $\lim_{n\to\infty} p_n\left(\frac{\lambda_0+\lambda_1+\lambda_2}{3}\right)$ does not exist.
4. $\lim_{n\to\infty} p_n'\left(\frac{\lambda_0+\lambda_1+\lambda_2}{3}\right)$ does not exist.

16. Define $f:\mathbb{R}^2 \to \mathbb{R}^2$ by $f(x,y) = (x+2y+y^2+|xy|, 2x+y+x^2+|xy|)$ for $(x,y)\in\mathbb{R}^2$. Then

1. f is discontinuous at (0,0).
2. f is continuous at (0,0) but not differentiable at (0,0).
3. f is differentiable at (0,0).
4. f is differentiable at (0,0) and the derivative Df(0,0) is invertible.

17. Let $A = \{(x,y)\in \mathbb{R}^2 : x+y \neq -1\}$ .Define $f: A \to \mathbb{R}^2$ by $f(x,y) = \left(\frac{x}{1+x+y}, \frac{y}{1+x+y}\right)$.

Then,

1. the Jacobian matrix of f does not vanish on A
2. f is infinitely differentiable on A
3. f is injective on A
4. f(A) $=\mathbb{R}^2$

18. Which of the following are compact?

1. $\{(x,y) \in \mathbb{R}^2 : (x-1)^2 + (y-2)^2 = 9\} \cup \{(x,y)\in\mathbb{R}^2 : y = 3\}$.
2. $\left\{\left(\frac{1}{m},\frac{1}{n}\right)\in \mathbb{R}^2 : m, n \in \mathbb{Z}\setminus\{0\}\right\} \cup \{(0,0)\} \cup \left\{\left(\frac{1}{m},0\right) : m \in \mathbb{Z}\setminus\{0\}\right\} \cup \left\{\left(0,\frac{1}{n}\right) : n\in\mathbb{Z}\setminus\{0\}\right\}$
3. $\{(x,y,z)\in\mathbb{R}^3 : x^2+2y^2-3z^2 = 1\}$.
4. $\{(x,y,z)\in\mathbb{R}^3 : |x|+2|y|+3|z| \le 1\}$.

DECEMBER – 2014

PART - B

19. Let $\{b_n\}$ and $\{c_n\}$ be sequences of real numbers. Then a necessary and sufficient condition for the sequence of polynomials $f_n(x) = b_n x + c_n x^2$ to converge uniformly to 0 on the real line is

1. $\lim_{n\to\infty} b_n = 0$ and $\lim_{n\to\infty} c_n = 0$
2. $\sum_{n=1}^{\infty} |b_n| < \infty$ and $\sum_{n=1}^{\infty} |c_n| < \infty$
3. There exists a positive integer N such that $b_n=0$ and $c_n=0$ for all n > N
4. $\lim_{n\to\infty} c_n = 0$

20. Let k be a positive integer. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} z^n$ is

1. k
2. k^{-k}
3. k^k
4. ∞

21. Suppose p is a polynomial with real coefficients. Then which of the following statements is necessarily true?

1. There is no root of the derivative p' between two real roots of the polynomial p.
2. There is exactly one root of the derivative p' between any two real roots of p.
3. There is exactly one root of the derivative p' between any two consecutive roots of p.
4. There is at least one root of the derivative p' between any two consecutive roots of p.

22. Let $G = \{(x, f(x)) : 0 \le x \le 1\}$ be the graph of a real valued differentiable function f. Assume that $(1,0) \in G$. Suppose that the tangent vector to G at any point is perpendicular to the radius vector at that point. Then which of the following is true?

1. G is the arc of an ellipse.
2. G is the arc of a circle.
3. G is a line segment.
4. G is the arc of a parabola.

23. Let $\Omega \subseteq \mathbb{R}^n$ be an open set and $f : \Omega \to \mathbb{R}$ be a differentiable function such that (Df)(x)=0 for all $x \in \Omega$. Then which of the following is true?

1. f must be a constant function.
2. f must be constant on connected components of Ω.
3. f(x)=0 or 1 for $x \in \Omega$.
4. The range of the function f is a subset of $\mathbb{Z}$.

24. Let $\{a_n : n \ge 1\}$ be a sequence of real numbers such that $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} |a_n|$ is divergent. Let R be the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$. Then we can conclude that

1. $0 < R < 1$
2. R=1
3. $1 < R < \infty$
4. R= ∞

25. Let P: $\mathbb{R}\to\mathbb{R}$ be a polynomial of the form $P(x)=a_0+a_1x+a_2x^2$, with $a_0,a_1,a_2 \in \mathbb{R}$ and $a_2 \ne 0$.

Let $E_1 = \int_0^1 P(x)dx - \frac{1}{2}(P(0)+P(1))$, $E_2 = \int_0^1 P(x)dx - P\left(\frac{1}{2}\right)$. If |x| is the absolute value of $x \in \mathbb{R}$, then

1. $|E_1|>|E_2|$
2. $|E_2|>|E_1|$
3. $|E_2|=|E_1|$
4. $|E_2|=2|E_1|$

PART – C

26. Let E be a subset of $\mathbb{R}$. Then the characteristic function $\chi_E : \mathbb{R}\to\mathbb{R}$ is continuous if and only if

1. E is closed
2. E is open
3. E is both open and closed
4. E is neither open nor closed

27. Suppose that P is a monic polynomial of degree n in one variable with real coefficients and K is a real number. Then which of the following statements is/are necessarily true?

1. If n is even and K >0, then there exists $x_0 \in \mathbb{R}$ such that P(x_0)=K e^{x_0}
2. If n is odd and K < 0, then there exists $x_0 \in \mathbb{R}$ such that P(x_0)=K e^{x_0}
3. For any natural number n and 0< K<1, there exists $x_0 \in \mathbb{R}$ such that $P(x_0) = Ke^{x_0}$
4. If n is odd and $K \in \mathbb{R}$, then there exists $x_0 \in \mathbb{R}$ such that $P(x_0) = Ke^{x_0}$

28. Let {a_k} be an unbounded, strictly increasing sequence of positive real numbers and $x_k = (a_{k+1} - a_k)/a_{k+1}$. Which of the following statements is/are correct?

1. For all $n \geq m, \sum_{k=m}^{n} x_k > 1 - \frac{a_m}{a_n}$.
2. There exists n ≥ m such that $\sum_{k=m}^{n} x_k > \frac{1}{2}$.
3. $\sum_{k=1}^{\infty} x_k$ converges to a finite limit.
4. $\sum_{k=1}^{\infty} x_k$ diverges to ∞.

29. For a non – empty subset S and a point x in a connected metric space (X,d), let d(x,S)=inf {d(x,y): $y \in S\}$. Which of the following statements is/are correct?

1. If S is closed and d(x,S)>0 then x is not an accumulation point of S
2. If S is open and d(x,S)>0 then x is not an accumulation point of S.
3. If S is closed and d(x,S)>0 then S does not contain x
4. If S is open and d(x,S)=0 then $x \in S$.

30. Let f be a continuously differentiable function on ℝ. Suppose that $L = \lim_{x\to\infty}(f(x) + f'(x))$ exists. If 0<L<∞, then which of the following statements is/are correct?

1. If $\lim_{x\to\infty} f'(x)$ exists, then it is 0
2. If $\lim_{x\to\infty} f(x)$ exists, then it is L
3. If $\lim_{x\to\infty} f'(x)$ exists, then $\lim_{x\to\infty} f(x) = 0$
4. If $\lim_{x\to\infty} f(x)$ exists, then $\lim_{x\to\infty} f'(x) = L$

31. Let A be a subset of ℝ. Which of the following properties imply that A is compact?

1. Every continuous function f from A to ℝ is bounded.
2. Every sequence {x_n} in A has a convergent subsequence converging to a point in A.
3. There exists a continuous function from A onto [0,1].
4. There is no one-one and continuous function from A onto (0,1).

32. Let f be a monotonically increasing function from [0,1] into [0,1]. Which of the following statements is/are true?

1. f must be continuous at all but finitely many points in [0,1].
2. f must be continuous at all but countably many points in [0,1].
3. f must be Riemann integrable.
4. f must be Lebesgue integrable.

33. Let X be a metric space and $f : X \to \mathbb{R}$ be a continuous function. Let G = {(x, f(x)): $x \in X\}$ be the graph of f. Then

1. G is homeomorphic to X

2. G is homeomorphic to $\mathbb{R}$
3. G is homeomorphic to $X \times \mathbb{R}$
4. G is homeomorphic to $\mathbb{R} \times X$

34. Let $X = \{(a,b) \in \mathbb{R}^2 : a^2 + b^2 = 1\}$ be the unit circle inside $\mathbb{R}^2$. Let $f : X \to \mathbb{R}$ be a continuous function. Then:
1. Image (f) is connected.
2. Image (f) is compact.
3. The given information is not sufficient to determine whether image (f) is bounded.
4. f is not injective.

JUNE – 2015

PART – B

35. The sum of the series $\frac{1}{1!} + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \ldots$ equals

1. e 2. $\frac{e}{2}$ 3. $\frac{3e}{2}$ 4. $1 + \frac{e}{2}$

36. The limit $\lim\limits_{x \to 0} \frac{1}{x} \int_x^{2x} e^{-t^2} dt$

1. does not exist.
2. is infinite.
3. exists and equals 1.
4. exists and equals 0.

37. Let $f : X \to X$ such that $f(f(x)) = x$ *for all* $x \in X$. Then
1. f is one to-one and onto.
2. f is one to-one ,but not onto.
3. f is onto but not one-to-one.
4. f need not be either one-to-one or onto.

38. A polynomial of odd degree with real coefficients must have
1. at least one real root.
2. no real root.
3. only real roots
4. at least one root which is not real.

39. Let for each n ≥ 1, C_n be the open disc in $\mathbb{R}^2$, with centre at the point (n,0) and radius equal to n. Then $C = \bigcup_{n \geq 1} C_n$ is
1. $\{(x,y) \in \mathbb{R}^2 : x > 0 \text{ and } |y| < x\}$
2. $\{(x,y) \in \mathbb{R}^2 : x > 0 \text{ and } |y| < 2x\}$
3. $\{(x,y) \in \mathbb{R}^2 : x > 0 \text{ and } |y| < 3x\}$
4. $\{(x,y) \in \mathbb{R}^2 : x > 0 \}$

40. Let f: $\mathbb{R} \to \mathbb{R}$ be a polynomial of the form $f(x) = a_0 + a_1 x + a_2 x^2$ with $a_0, a_1, a_2 \in \mathbb{R}$ *and* $a_2 \neq 0$. If $E_1 = \int_{-1}^{1} f(x)dx - [f(-1) + f(1)]$, $E_2 = \int_{-1}^{1} f(x)dx - \frac{1}{2}(f(-1) + 2f(0) + f(1))$ and $|x|$ is the absolute value of $x \in \mathbb{R}$, then

1. $|E_1| < |E_2|$ 2. $|E_1| = 2|E_2|$ 3. $|E_1| = 4|E_2|$ 4. $|E_1| = 8|E_2|$

PART – C

41. Let a be positive real number. Which of the following integrals are convergent?

1. $\int_0^a \frac{1}{x^4}dx$ 2. $\int_0^a \frac{1}{\sqrt{x}}dx$ 3. $\int_4^\infty \frac{1}{x\log_e x}dx$ 4. $\int_5^\infty \frac{1}{x(\log_e x)^2}dx$

42. For n ≥ 1, let $g_n(x) = \sin^2\left(x + \frac{1}{n}\right), x \in [0,\infty)$ and $f_n(x) = \int_0^x g_n(t)dt.$ Then

1. $\{f_n\}$ converges pointwise to a function f on $[0,\infty)$, but does not converge uniformly on $[0,\infty)$.
2. $\{f_n\}$ does not converge pointwise to any function on $[0,\infty)$.
3. $\{f_n\}$ converges uniformly on [0,1].
4. $\{f_n\}$ converges uniformly on $[0,\infty)$.

43. Which of the following sets in $\mathbb{R}^2$ have positive Lebesgue measure?

For two sets A, B $\subseteq \mathbb{R}^2$,
A + B = {a + b | a $\in$ A, b $\in$ B}

1. S = {(x,y)| $x^2 + y^2 = 1$}
2. S = {(x,y)| $x^2 + y^2 < 1$}
3. S = {(x,y)| x = y} + {(x,y)| x = - y}
4. S = {(x,y)| x = y} + {(x,y)| x = y}

44. Let f be a bounded function on $\mathbb{R}$ and $a \in \mathbb{R}$. For δ > 0, Let ω(a, δ) = sup|f(x) – f(a)|, x ∈ [a-δ, a+δ]. Then

1. $\omega(a,\delta_1) \le \omega(a,\delta_2)$ if $\delta_1 \le \delta_2$
2. $\lim_{\delta\to 0+} \omega(a,\delta) = 0$ *for all* $a \in \mathbb{R}$
3. $\lim_{\delta\to 0+} \omega(a,\delta)$ need not exist.
4. $\lim_{\delta\to 0+} \omega(a,\delta) = 0$ if and only if f is continuous at a.

45. For n ≥ 2, let $a_n = \frac{1}{n\log n}$. Then

1. The sequence $\{a_n\}_{n=2}^\infty$ is convergent.
2. The series $\sum_{n=2}^\infty a_n$ is convergent.
3. The series $\sum_{n=2}^\infty a_n^2$ is convergent.
4. The series $\sum_{n=2}^\infty (-1)^n a_n$ is convergent.

46. Which of the following sets of functions are uncountable? (ℕ stands for the set of natural numbers.)

1. { f | f: ℕ → {1,2}}
2. { f | f: {1,2} → ℕ}
3. { f | f: {1,2} → ℕ, f(1) ≤ f(2)}
4. { f | f: ℕ → {1,2}, f(1) ≤ f(2)}

47. Let $\{a_0, a_1, a_2, ...\}$ be a sequence of real numbers. For any k ≥ 1, let $s_n = \sum_{k=0}^n a_{2k}$. Which of the following statements are correct?

1. If $\lim_{n\to\infty} s_n$ exists, then $\sum_{m=0}^\infty a_m$ exists.
2. If $\lim_{n\to\infty} s_n$ exists, then $\sum_{m=0}^\infty a_m$ need not exist.
3. If $\sum_{m=0}^\infty a_m$ exists, then $\lim_{n\to\infty} s_n$ exists.

4. If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} s_n$ need not exist.

DECEMBER- 2015

PART - B

48. For (x,y) $\in \mathbb{R}^2$ with (x, y) $\neq$ (0,0), let $\theta = \theta$ (x,y) be the unique real number such that $-\pi < \theta \leq \pi$ and (x,y) = (r cos θ, r sin θ), where $r = \sqrt{x^2 + y^2}$. Then the resulting function $\theta : \mathbb{R}^2 \backslash \{(0,0)\} \to \mathbb{R}$ is

1. differentiable
2. continuous, but not differentiable
3. bounded, but not continuous
4. neither bounded, nor continuous

49. Let f : $\mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function, with f(0) = f(1) = f ′(0) = 0. Then

1. f ″ is the zero function.
2. f ″(0) is zero.
3. f ″(x) = 0 for some x $\in$ (0,1)
4. f ″ never vanishes.

50. $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{2}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{6}} + ... + \frac{1}{\sqrt{2n}+\sqrt{2n+2}}\right)$ is

1. $\sqrt{2}$
2. $\frac{1}{\sqrt{2}}$
3. $\sqrt{2}+1$
4. $\frac{1}{\sqrt{2}+1}$

51. Let $S_n = \sum_{k=1}^{n} \frac{1}{k}$. Which of the following is true?

1. $S_{2^n} \geq \frac{n}{2}$ for every n ≥ 1.
2. S_n is a bounded sequence
3. $\left|S_{2^n} - S_{2^{n-1}}\right| \to 0$ as n $\to \infty$
4. $\frac{S_n}{n} \to 1$ as n $\to \infty$

52. Let A be a closed subset of $\mathbb{R}$, A $\neq \phi$, A $\neq \mathbb{R}$. Then A is

1. the closure of the interior of A.
2. a countable set
3. a compact set
4. not open

53. Let f : [0, ∞) → [0, ∞) be a continuous function. Which of the following is correct?

1. There is $x_0 \in [0, \infty)$ such that $f(x_0) = x_0$.
2. If f(x) ≤ M for all x $\in$ [0, ∞) for some M > 0, then there exists $x_0 \in [0, \infty)$ such that $f(x_0) = x_0$
3. If f has a fixed point, then it must be unique
4. f does not have a fixed point unless it is differentiable on (0, ∞)

PART – C

54. Let f : $\mathbb{R} \to \mathbb{R}$ be a differentiable function such that $\sup_{x\in R} |f'(x)| < \infty$. Then,

1. f maps a bounded sequence to a bounded sequence.
2. f maps a Cauchy sequence to a Cauchy sequence.
3. f maps a convergent sequence to a convergent sequence.
4. f is uniformly continuous.

55. For (x,y) $\in \mathbb{R}^2$, consider the series $\lim_{n\to\infty} \sum_{\ell,k=0}^{n} \frac{k^2 x^k y^\ell}{\ell!}$. Then the series converges for (x,y) in

1. $(-1, 1) \times (0, \infty)$ 2. $\mathbb{R} \times (-1, 1)$
3. $(-1, 1) \times (-1, 1)$ 4. $\mathbb{R} \times \mathbb{R}$

56. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by the formula $f(x,y) = (3x + 2y + y^2 + |xy|, 2x + 3y + x^2 + |xy|)$. Then,
1. f is discontinuous at (0,0).
2. f is continuous at (0,0) but not differentiable at (0,0).
3. f is differentiable at (0,0).
4. f is differentiable at (0,0) and the derivative Df (0,0) is invertible.

57. Let $p_n(x) = a_n x^2 + b_n x$ be a sequence of quadratic polynomials where $a_n, b_n \in \mathbb{R}$ for all $n \geq 1$. Let λ_0, λ_1 be distinct non-zero real numbers such that $\lim_{n\to\infty} p_n(\lambda_0)$ and $\lim_{n\to\infty} p_n(\lambda_1)$ exist. Then,
1. $\lim_{n\to\infty} p_n(x)$ exists for all $x \in \mathbb{R}$. 2. $\lim_{n\to\infty} p'_n(x)$ exists for all $x \in \mathbb{R}$.
3. $\lim_{n\to\infty} p_n\left(\frac{\lambda_0 + \lambda_1}{2}\right)$ does not exist. 4. $\lim_{n\to\infty} p'_n\left(\frac{\lambda_0 + \lambda_1}{2}\right)$ does not exist.

58. Let t and a be positive real numbers. Define $B_a = \{x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n, x_1^2 + x_2^2 + \ldots + x_n^2 \leq a^2\}$.
Then for any compactly supported continuous function f on $\mathbb{R}^n$ which of the following are correct?
1. $\int_{B_a} f(tx)\,dx = \int_{B_{ta}} f(x)\,t^{-n}\,dx$. 2. $\int_{B_a} f(tx)\,dx = \int_{B_{t^n a}} f(x)\,t\,dx$.
3. $\int_{\mathbb{R}^n} f(x + y)\,dx = \int_{\mathbb{R}^n} f(x)\,dx$, for some $y \in \mathbb{R}^n$. 4. $\int_{\mathbb{R}^n} f(tx)\,dx = \int_{\mathbb{R}^n} f(x)\,t^n\,dx$.

59. Consider all sequences $\{f_n\}$ of real valued continuous functions on $[0, \infty)$. Identify which of the following statements are correct.
1. If $\{f_n\}$ converges to f pointwise on $[0, \infty)$, then $\lim_{n\to\infty} \int_0^\infty f_n(x)\,dx = \int_0^\infty f(x)\,dx$
2. If $\{f_n\}$ converges to f uniformly on $[0, \infty)$, then $\lim_{n\to\infty} \int_0^\infty f_n(x)\,dx = \int_0^\infty f(x)\,dx$
3. If $\{f_n\}$ converges to f uniformly on $[0, \infty)$, then f is continuous on $[0, \infty)$.
4. There exists a sequence of continuous functions $\{f_n\}$ on $[0, \infty)$ such that $\{f_n\}$ converges to f uniformly on $[0, \infty)$ but $\lim_{n\to\infty} \int_0^\infty f_n(x)\,dx \neq \int_0^\infty f(x)\,dx.$

60. Let G_1 and G_2 be two subsets of $\mathbb{R}^2$ and $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a function. Then,
1. $f^{-1}(G_1 \cup G_2) = f^{-1}(G_1) \cup f^{-1}(G_2)$
2. $f^{-1}(G_1^c) = (f^{-1}(G_1))^c$
3. $f(G_1 \cap G_2) = f(G_1) \cap f(G_2)$
4. If G_1 is open and G_2 is closed then $G_1 + G_2 = \{x + y : x \in G_1, y \in G_2\}$ is neither open nor closed.

61. Let $A = \{(x,y) \in \mathbb{R}^2 : x+y \neq -1\}$. Define $f:A \to \mathbb{R}^2$ by $f(x,y) = \left(\frac{y}{1+x+y}, \frac{x}{1+x+y}\right)$. Then,
1. the determinant of the Jacobian of f does not vanish on A.
2. f is infinitely differentiable on A.
3. f is one to one.
4. $f(A) = \mathbb{R}^2$.

62. Let f : $\mathbb{R}^2 \to \mathbb{R}^2$ be the function f(r, θ) = (r cosθ, r sinθ). Then for which of the open subsets U of $\mathbb{R}^2$ given below, f restricted to U admits an inverse?

1. U = $\mathbb{R}^2$
2. U = {(x,y) ∈ $\mathbb{R}^2$: x > 0, y > 0}
3. U = {(x,y) ∈ $\mathbb{R}^2$: $x^2 + y^2 < 1$}
4. U = {(x,y) ∈ $\mathbb{R}^2$: x < -1, y < -1}

63. Let S ⊂ $\mathbb{R}^2$ be defined by S = $\left\{ \left(m + \frac{1}{4^{|p|}}, n + \frac{1}{4^{|q|}} \right) : \text{m, n, p, q} \in \mathbb{Z} \right\}$. Then,

1. S is discrete in $\mathbb{R}^2$.
2. The set of limit points of S is the set {(m, n):m, n ∈ $\mathbb{Z}$}.
3. S^c is connected but not path connected.
4. S^c is path connected.

64. Which of the following statements is/are true?

1. There exists a continuous map f : $\mathbb{R} \to \mathbb{R}$ such that f ($\mathbb{R}$) = $\mathbb{Q}$.
2. There exists a continuous map f : $\mathbb{R} \to \mathbb{R}$ such that f($\mathbb{R}$) = $\mathbb{Z}$.
3. There exists a continuous map f : $\mathbb{R} \to \mathbb{R}^2$ such that f($\mathbb{R}$) = {(x,y) ∈ $\mathbb{R}^2$: $x^2 + y^2 = 1$}.
4. There exists a continuous map f: [0,1] ∪ [2,3] → {0,1}.

65. Let f : (0, 1) → $\mathbb{R}$ be continuous. Suppose that |f(x) – f(y)| ≤ |cos x – cos y| for all x, y ∈ (0, 1). Then,

1. f is discontinuous at least at one point in (0, 1).
2. f is continuous everywhere on (0, 1) but not uniformly continuous on (0, 1).
3. f is uniformly continuous on (0, 1).
4. $\lim_{x \to 0^+}$ f(x) exists.

JUNE – 2016

PART – B

66. Consider the improper Riemann integral $\int_0^x y^{-1/2} dy$. This integral is:

1. continuous in [0, ∞)
2. continuous only in (0, ∞)
3. discontinuous in (0, ∞)
4. discontinuous only in (1/2, ∞)

67. Which one of the following statements is true for the sequence of functions

$$f_n(x) = \frac{1}{n^2 + x^2}, n = 1, 2, ..., x \in [1/2, 1]?$$

1. The sequence is monotonic and has 0 as the limit for all x ∈ [1/2, 1] as $n \to \infty$.
2. The sequence is not monotonic but has $f(x) = \frac{1}{x^2}$ as the limit as $n \to \infty$.
3. The sequence is monotonic and has $f(x) = \frac{1}{x^2}$ as the limit as $n \to \infty$.
4. The sequence is not monotonic but has 0 as the limit.

68. $\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^n$ equals

1. 1 2. $e^{-1/2}$ 3. e^{-2} 4. e^{-1}

69. Consider the interval (-1,1) and a sequence $\{\alpha_n\}_{n=1}^{\infty}$ of elements in it. Then,

1. Every limit point of $\{\alpha_n\}$ is in (-1,1)
2. Every limit point of $\{\alpha_n\}$ is in [-1,1]
3. The limit points of $\{\alpha_n\}$ can only be in {-1,0,1}
4. The limit points of $\{\alpha_n\}$ cannot be in {-1,0,1}

70. Let F : $\mathbb{R}\to\mathbb{R}$ be a monotonic function. Then
1. F has no discontinuities
2. F has only finitely many discontinuities
3. F can have at most countably many discontinuities
4. F can have uncountably many discontinuities

71. Consider the function $f(x,y)=\frac{x^2}{y^2},(x,y)\in[1/2,3/2]\times[1/2,3/2].$

The derivative of the function at (1,1) along the direction (1,1) is
1. 0 2. 1 3. 2 4. –2

PART – C

72. Let $\{x_n\}$ be an arbitrary sequence of real numbers. Then
1. $\sum_{n=1}^{\infty}|x_n|^p<\infty$ for some $1 < p < \infty$ implies $\sum_{n=1}^{\infty}|x_n|^q<\infty$ for q > p.
2. $\sum_{n=1}^{\infty}|x_n|^p<\infty$ for some $1 < p < \infty$ implies $\sum_{n=1}^{\infty}|x_n|^q<\infty$ for any $1 \le q < p$.
3. Given any $1<p<q<\infty$, there is a real sequence $\{x_n\}$ such that $\sum_{n=1}^{\infty}|x_n|^p<\infty$ but $\sum_{n=1}^{\infty}|x_n|^q=\infty.$
4. Given any $1<q<p<\infty$, there is a real sequence $\{x_n\}$ such that $\sum_{n=1}^{\infty}|x_n|^p<\infty$ but $\sum_{n=1}^{\infty}|x_n|^q=\infty.$

73. Let f : $\mathbb{R}\to\mathbb{R}$ be a continuous function and f(x+1) = f(x) for all $x\in\mathbb{R}$. Then
1. f is bounded above, but not bounded below
2. f is bounded above and below, but may not attain its bounds.
3. f is bounded above and below and f attains its bounds.
4. f is uniformly continuous.

74. Let $x_1 = 0$, $x_2 = 1$, and for $n \ge 3$, define $x_n=\frac{x_{n-1}+x_{n-2}}{2}$. Which of the following is/are true?

1. $\{x_n\}$ is a monotone sequence. 2. $\lim_{n\to\infty} x_n=\frac{1}{2}.$

3. $\{x_n\}$ is a Cauchy sequence. 4. $\lim_{n\to\infty} x_n=\frac{2}{3}.$

75. Take the closed interval [0,1] and open interval (1/3, 2/3). Let K = [0,1]\(1/3,2/3). For $x \in [0,1]$ define f(x) = d(x,K) where d(x,K) = inf {|x – y| | y ∈ K}. Then

1. f : $[0,1] \to \mathbb{R}$ is differentiable at all points of (0,1)
2. f: $[0,1] \to \mathbb{R}$ is not differentiable at 1/3 and 2/3
3. f: $[0,1] \to \mathbb{R}$ is not differentiable at 1/2
4. f: $[0,1] \to \mathbb{R}$ is not continuous

76. Which of the following functions is/are uniformly continuous on the interval (0,1) ?

1. $\frac{1}{x}$ 2. $\sin\frac{1}{x}$ 3. $x\sin\frac{1}{x}$ 4. $\frac{\sin x}{x}$

77. Let A be any set. Let $\mathbb{P}(A)$ be the power set of A, that is, the set of all subsets of A; $\mathbb{P}(A)=\{B : B \subseteq A\}$.

Then which of the following is/are true about the set $\mathbb{P}(A)$?

1. $\mathbb{P}(A)= \phi$ for some A.
2. $\mathbb{P}(A)$ is a finite set for some A.
3. $\mathbb{P}(A)$ is a countable set for some A.
4. $\mathbb{P}(A)$ is a uncountable set for some A.

78. Define f on [0,1] by $f(x)=\begin{cases} x^2 & if\ \ x\ is\ rational \\ x^3 & if\ \ x\ is\ irrational \end{cases}$. Then

1. f is not Riemann integrable on [0,1].
2. f is Riemann integrable and $\int_0^1 f(x)dx=\frac{1}{4}$.
3. f is Riemann integrable and $\int_0^1 f(x)dx=\frac{1}{3}$.
4. $\frac{1}{4}=\underline{\int_0^1} f(x)dx<\overline{\int_0^1} f(x)dx=\frac{1}{3}$, where $\underline{\int_0^1} f(x)dx$ and $\overline{\int_0^1} f(x)dx$ are the lower and upper Riemann integrals of f.

79. Consider the integral $A=\int_0^1 x^n(1-x)^n\,dx.$ Pick each correct statement from below.

1. A is not a rational number
2. $0 < A \le 4^{-n}$.
3. A is a natural number.
4. A^{-1} is a natural number.

DECEMBER - 2016

PART – B

80. Consider the sets of sequences X = {(x_n) : $x_n \in \{0, 1\}$, $n \in \mathbb{N}$} and Y = {$(x_n) \in X$: $x_n = 1$ for at most finitely many n}.Then

1. X is countable, Y is finite.
2. X is uncountable, Y is countable.
3. X is countable, Y is countable.
4. X is uncountable, Y is uncountable.

81. Let f: $\mathbb{R}^2 \to \mathbb{R}^2$ be given by f(x, y) = $(x^2, y^2 + \sin x)$. Then the derivative of f at (x, y) is the linear transformation given by

1. $\begin{pmatrix} 2x & 0 \\ \cos x & 2y \end{pmatrix}$ 2. $\begin{pmatrix} 2x & 0 \\ 2y & \cos x \end{pmatrix}$ 3. $\begin{pmatrix} 2y & \cos x \\ 2x & 0 \end{pmatrix}$ 4. $\begin{pmatrix} 2x & 2y \\ 0 & \cos x \end{pmatrix}$

82. A function $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x, y) = xy$. Let $v = (1, 2)$ and $a = (a_1, a_2)$ be two elements of $\mathbb{R}^2$. The directional derivative of f in the direction of v at a is:

1. $a_1 + 2a_2$
2. $a_2 + 2a_1$
3. $\frac{a_2}{2} + a_1$
4. $\frac{a_1}{2} + a_2$

83. $\lim_{n\to\infty} \frac{1}{n^4} \sum_{j=0}^{2n-1} j^3$ equals

1. 4
2. 16
3. 1
4. 8

84. $f : \mathbb{R} \to \mathbb{R}$ is such that $f(0) = 0$ and $\left|\frac{df}{dx}(x)\right| \le 5$ for all x. We can conclude that f(1) is in

1. (5, 6)
2. [-5, 5]
3. $(-\infty, -5) \cup (5, \infty)$
4. [-4, 4]

85. Let G be an open set in $\mathbb{R}^n$. Two points $x, y \in G$ are said to be equivalent if they can be joined by a continuous path completely lying inside G. Number of equivalence classes is

1. only one
2. at most finite
3. at most countable
4. can be finite, countable or uncountable

PART – C

86. Let $s \in (0,1)$. Then decide which of the following are true.

1. $\forall m \in \mathbb{N}$, $\exists\, n \in \mathbb{N}$ s.t. $s > m/n$
2. $\forall m \in \mathbb{N}$, $\exists\, n \in \mathbb{N}$ s.t. $s < m/n$
3. $\forall m \in \mathbb{N}$, $\exists\, n \in \mathbb{N}$ s.t. $s = m/n$
4. $\forall m \in \mathbb{N}$, $\exists\, n \in \mathbb{N}$ s.t. $s = m+n$

87. Let $f_n(x) = (-x)^n$, $x \in [0,1]$. Then decide which of the following are true.

1. there exists a pointwise convergent subsequence of f_n.
2. f_n has no pointwise convergent subsequence.
3. f_n converges pointwise everywhere.
4. f_n has exactly one pointwise convergent subsequence.

88. Which of the following are true for the function $f(x) = \sin(x)\sin\left(\frac{1}{x}\right)$, $x \in (0,1)$?

1. $\underline{\lim}_{x\to 0} f(x) = \overline{\lim}_{x\to 0} f(x)$
2. $\underline{\lim}_{x\to 0} f(x) < \overline{\lim}_{x\to 0} f(x)$
3. $\underline{\lim}_{x\to 0} f(x) = 1$
4. $\overline{\lim}_{x\to 0} f(x) = 0$

89. Find out which of the following series converge uniformly for $x \in (-\pi, \pi)$.

1. $\sum_{n=1}^{\infty} \frac{e^{-n|x|}}{n^3}$
2. $\sum_{n=1}^{\infty} \frac{\sin(xn)}{n^5}$
3. $\sum_{n=1}^{\infty} \left(\frac{x}{n}\right)^n$
4. $\sum_{n=1}^{\infty} \frac{1}{((x+\pi)n)^2}$

90. Decide which of the following functions are uniformly continuous on (0,1).

1. $f(x) = e^x$
2. $f(x) = x$
3. $f(x) = \tan\left(\frac{\pi x}{2}\right)$
4. $f(x) = \sin(x)$

91. Let $\chi_A(x)$ denote the function which is 1 if x ∈ A and 0 otherwise. Consider $f(x)=\sum_{n=1}^{200}\frac{1}{n^6}\chi_{\left[0,\frac{n}{200}\right]}(x),\ x\in[0,1].$ Then f(x) is

1. Riemann integrable on [0,1].
2. Lebesgue integrable on [0,1]
3. is a continuous function on [0,1].
4. is a monotone function on [0,1].

92. A function f(x,y) on $\mathbb{R}^2$ has the following partial derivatives $\frac{\partial f}{\partial x}(x,y)=x^2, \frac{\partial f}{\partial y}(x,y)=y^2.$ Then

1. f has directional derivatives in all directions everywhere.
2. f has derivative at all points.
3. f has directional derivative only along the direction (1,1) everywhere.
4. f does not have directional derivatives in any direction everywhere.

93. Let d_1, d_2 be the following metrices on $\mathbb{R}^n$. $d_1(x,y)=\sum_{i=1}^{n}|x_i-y_i|,\ d_2(x,y)=\left(\sum_{i=1}^{n}|x_i-y_i|^2\right)^{\frac{1}{2}}$. Then decide which of the following is a metric on $\mathbb{R}^n$?

1. $d(x,y)=\frac{d_1(x,y)+d_2(x,y)}{1+d_1(x,y)+d_2(x,y)}$
2. $d(x,y)=d_1(x,y)-d_2(x,y)$
3. $d(x,y)=d_1(x,y)+d_2(x,y)$
4. $d(x,y)=e^{\pi}d_1(x,y)+e^{-\pi}d_2(x,y)$

94. Let A be the following subset of $\mathbb{R}^2$:A = {(x,y):$(x+1)^2+y^2\le 1$}∪{(x,y):y=x sin $\frac{1}{x}, x>0$}. Then

1. A is connected
2. A is compact
3. A is path connected
4. A is bounded

95. Let (X,d) be a metric space. Then

1. An arbitrary open set G in X is a countable union of closed sets.
2. An arbitrary open set G in X cannot be countable union of closed sets if X is connected.
3. An arbitrary open set G in X is a countable union of closed sets only if X is countable.
4. An arbitrary open set G in X is a countable union of closed sets only if X is locally compact.

96. Let S = {(x,y)∈$\mathbb{R}^2$| -1 ≤ x ≤ 1 and -1 ≤ y ≤ 1}. Let T=S\ (0,0), the set obtained by removing the origin from S. Let f be a continuous function from T to ℝ. Choose all correct options.

1. Image of f must be connected.
2. Image of f must be compact.
3. Any such continuous function f can be extended to a continuous function from S to ℝ.
4. If f can be extended to a continuous function from S to ℝ then the image of f is bounded.

JUNE – 2017

PART - B

97. $L=\lim_{n\to\infty}\frac{1}{\sqrt[n]{n!}}$. Then

1. $L=0$
2. $L=1$
3. $0<L<\infty$
4. $L=\infty$

98. Consider the sequence $a_n = \left(1 + (-1)^n \frac{1}{n}\right)^n$. Then

1. $\limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n = 1$
2. $\limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n = e$
3. $\limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n = \frac{1}{e}$
4. $\limsup_{n\to\infty} a_n = e, \liminf_{n\to\infty} a_n = \frac{1}{e}$

99. For $a > 0$, the series $\sum_{n=1}^{\infty} a^{\ell n\, n}$ is convergent if and only if

1. $0 < a < e$
2. $0 < a \le e$
3. $0 < a < \frac{1}{e}$
4. $0 < a \le \frac{1}{e}$

100. Let f : ℝ → ℝ be defined by $f(x) = \begin{cases} \frac{\sin x}{x} & if \quad x \ne 0 \\ 1 & if \quad x = 0 \end{cases}$.Then

1. f is not continuous
2. f is continuous but not differentiable
3. f is differentiable
4. f is not bounded

101. Let A = {n ∈ ℕ : n = 1 or the only prime factors of n are 2 or 3}, for example, 6 ∈ A, 10 ∉ A. Let $S = \sum_{n\in A} \frac{1}{n}$. Then

1. A is finite
2. S is a divergent series
3. S = 3
4. S = 6

102. For n ≥ 1, let $f_n(x) = xe^{-nx^2}$, x ∈ ℝ. Then the sequence $\{f_n\}$ is

1. uniformly convergent on ℝ
2. uniformly convergent only on compact subsets of ℝ
3. bounded and not uniformly convergent on ℝ
4. a sequence of unbounded functions

103. Let f : ℝ → ℝ be a continuous map. Choose the correct statement.

1. f is bounded
2. The image of f is an open subset of ℝ
3. $f(A)$ is bounded for all bounded subsets A of ℝ
4. $f^{-1}(A)$ is compact for all compact subsets A of ℝ

104. Suppose $x:[0,\infty)\to[0,\infty)$ is continuous and $x(0) = 0$. If $(x(t))^2 \le 2 + \int_0^t x(s)\,ds,\ \forall t \ge 0,$ then which of the following is TRUE?

1. $x(\sqrt{2}) \in [0,2]$
2. $x(\sqrt{2}) \in \left[0, \frac{3}{\sqrt{2}}\right]$
3. $x(\sqrt{2}) \in \left[\frac{5}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right]$
4. $x(\sqrt{2}) \in [10, \infty]$

PART - C

105. Let α = 0. 1 0 1 1 0 1 1 1 0 1 1 1 1 0 … be a given real number written in base 10, that is, the n-th digit of α is 1, unless n is of the form $\frac{k(k+1)}{2}-1$ in which case it is 0. Choose all the correct statements from below.

1. α is a rational number
2. α is an irrational number
3. For every integer q ≥ 2, there exists an integer r ≥ 1 such that $\frac{r}{q} < \alpha < \frac{r+1}{q}$.
4. α has no periodic decimal expansion.

106. For a,b $\in \mathbb{N}$, consider the sequence $d_n = \frac{\binom{n}{a}}{\binom{n}{b}}$ for n>a,b. Which of the following statements are true?

1. $\{d_n\}$ converges for all values of a and b
2. $\{d_n\}$ converges if a < b
3. $\{d_n\}$ converges if a = b
4. $\{d_n\}$ converges if a > b

107. Let $\{a_n\}$ be a sequence of real numbers satisfying $\sum_{n=1}^{\infty} |a_n - a_{n-1}| < \infty$. Then the series $\sum_{n=0}^{\infty} a_n x^n, x \in \mathbb{R}$ is convergent

1. nowhere on $\mathbb{R}$
2. everywhere on $\mathbb{R}$
3. on some set containing (-1,1)
4. only on (-1,1)

108. Let $f(x) = \tan^{-1} x, x \in \mathbb{R}$. Then

1. there exists a polynomial p(x) satisfies $p(x) f'(x) = 1,$ for all x
2. $f^{(n)}(0) = 0$ for all positive even integer n
3. The sequence $\{f^{(n)}(0)\}$ is unbounded
4. $f^{(n)}(0) = 0$ for all n

109. Let $f_n(x) = \frac{1}{1+n^2x^2}$ for n $\in \mathbb{N}$, x $\in \mathbb{R}$. Which of the following are true?

1. f_n converges pointwise on [0,1] to a continuous function
2. f_n converges uniformly on [0,1]
3. f_n converges uniformly on $\left[\frac{1}{2},1\right]$
4. $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 (\lim_{n\to\infty} f_n(x))dx$

110. If $\lambda_n = \int_0^1 \frac{dt}{(1+t)^n}$ for n $\in \mathbb{N}$, then

1. λ_n does not exist for some n

2. λ_n exists for every n and the sequence is unbounded
3. λ_n exists for every n and the sequence is bounded
4. $\lim_{n\to\infty}(\lambda_n)^{1/n} = 1$

111. The equation $11^x + 13^x + 17^x - 19^x = 0$ has
1. no real root
2. only one real root
3. exactly two real roots
4. more than two real roots

112. Suppose that $f : \mathbb{R}^n \to \mathbb{R}$ is given by $f(\underline{x}) = a_1x_1^2 + a_2x_2^2 + ... + a_nx_n^2$, where $\underline{x} = (x_1, x_2, ..., x_n)$ and at least one a_j is not zero. Then we can conclude that
1. f is not everywhere differentiable
2. the gradient $(\nabla f)(\underline{x}) \neq 0$ for every $\underline{x} \in \mathbb{R}^n$
3. If $\underline{x} \in \mathbb{R}^n$ is such that $(\nabla f)(\underline{x}) = 0$ then $f(\underline{x}) = 0$
4. If $x \in \mathbb{R}^n$ is such that $f(\underline{x}) = 0$ then $(\nabla f)(\underline{x}) = 0$

113. Let S be the set of $(\alpha,\beta) \in \mathbb{R}^2$ such that $\frac{x^\alpha y^\beta}{\sqrt{x^2+y^2}} \to 0$ as $(x,y) \to (0,0)$. Then S is contained in
1. $\{(\alpha,\beta) : \alpha > 0, \beta > 0\}$
2. $\{(\alpha,\beta) : \alpha > 2, \beta > 2\}$
3. $\{(\alpha,\beta) : \alpha + \beta > 1\}$
4. $\{(\alpha,\beta) : \alpha + 4\beta > 1\}$

DECEMBER - 2017

PART – B

114. Let $\mathbb{Z}$ denote the set of integers and $\mathbb{Z}_{\geq 0}$ denote the set {0, 1, 2, 3,…}. Consider the map $f : \mathbb{Z}_{\geq 0} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m, n) = 2^m.(2n + 1)$. Then the map f is
1. onto (surjective) but not one-one (injective)
2. one-one (injective) but not onto (surjective)
3. both one-one and onto
4. neither one-one nor onto

115. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers satisfying $a_1 \geq 1$ and $a_{n+1} \geq a_n + 1$ for all $n \geq 1$. Then which of the following is necessarily true?
1. The series $\sum_{n=1}^{\infty}\frac{1}{a_n^2}$ diverges
2. The sequence $\{a_n\}_{n\geq 1}$ is bounded
3. The series $\sum_{n=1}^{\infty}\frac{1}{a_n^2}$ converges
4. The series $\sum_{n=1}^{\infty}\frac{1}{a_n}$ converges

116. Let D be a subset of the real line. Consider the assertion: "Every infinite sequence in D has a subsequence which converges in D". This assertion is true if
1. D = [0, ∞)
2. D = [0, 1] ∪ [3, 4]
3. D = [-1, 1) ∪ (1, 2]
4. D = (-1, 1]

117. Let $f : (0, \infty) \to \mathbb{R}$ be uniformly continuous. Then

1. $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to\infty} f(x)$ exist.
2. $\lim_{x\to 0^+} f(x)$ exists but $\lim_{x\to\infty} f(x)$ need not exist.
3. $\lim_{x\to 0^+} f(x)$ need not exist but $\lim_{x\to\infty} f(x)$ exists.
4. neither $\lim_{x\to 0^+} f(x)$ nor $\lim_{x\to\infty} f(x)$ need exist.

118. Let S = {f : ℝ → ℝ| ∃ ∈ > 0 such that ∀ δ > 0, |x – y| < δ ⇒ |f(x) – f(y)| < ∈}. Then

1. S = {f : ℝ → ℝ | f is continuous}
2. S = {f : ℝ → ℝ | f is uniformly continuous}
3. S = {f : ℝ → ℝ | f is bounded}
4. S = {f : ℝ → ℝ | f is constant}

119. Which of the following is necessarily true for a function f : X → Y?
1. if f is injective, then there exists g : Y → X such that f(g(y)) = y for all y ∈ Y
2. if f is surjective, then there exists g :Y → X such that f(g(y)) = y for all y ∈ Y
3. if f is injective and Y is countable then X is finite
4. if f is surjective and X is uncountable then Y is countably infinite

120. Let k be a positive integer and let S_k = {x ∈ [0, 1] | a decimal expansion of x has a prime digit at its k^{th} place}. Then the Lebesgue measure of S_k is
1. 0
2. 4/10
3. $(4/10)^k$
4. 1

121. Let S = {x ∈ [-1, 4] | sin (x) > 0}. Which of the following is true?
1. inf (S) < 0
2. sup (S) does not exist
3. sup (S) = π
4. inf (S) = π/2

122. Let A be a connected open subset of $\mathbb{R}^2$. The number of continuous surjective functions from $\overline{A}$ (the closure of A in $\mathbb{R}^2$) to ℚ is:
1. 1
2. 0
3. 2
4. not finite

PART - C

123. Which of the following are convergent?
1. $\sum_{n=1}^{\infty} n^2 2^{-n}$
2. $\sum_{n=1}^{\infty} n^{-2} 2^{n}$
3. $\sum_{n=2}^{\infty} \frac{1}{n\log n}$
4. $\sum_{n=1}^{\infty} \frac{1}{n\log(1+1/n)}$

124. Let $a_{mn}, m \geq 1, n \geq 1$ be a double array of real numbers. Define

$$P = \liminf_{n\to\infty} \liminf_{m\to\infty} a_{mn},$$
$$Q = \liminf_{n\to\infty} \limsup_{m\to\infty} a_{mn},$$
$$R = \limsup_{n\to\infty} \liminf_{m\to\infty} a_{mn},$$
$$S = \limsup_{n\to\infty} \limsup_{m\to\infty} a_{mn}$$

Which of the following statements are necessarily true ?

1. P ≤ Q
2. Q ≤ R
3. R ≤ S
4. P ≤ S

125. Let $\mathbb{R}$ denote the set of real numbers and $\mathbb{Q}$ the set of all rational numbers. For $0 \le \varepsilon \le \frac{1}{2}$, let A_ϵ be the open interval $(0,1-\varepsilon)$. Which of the following are true?

1. $\sup_{0<\varepsilon<\frac{1}{2}} \sup(A_\varepsilon) < 1$
2. $0 < \varepsilon_1 < \varepsilon_2 < \frac{1}{2} \Rightarrow \inf(A_{\varepsilon_1}) < \inf(A_{\varepsilon_2})$
3. $0 < \varepsilon_1 < \varepsilon_2 < \frac{1}{2} \Rightarrow \sup(A_{\varepsilon_1}) > \sup(A_{\varepsilon_2})$
4. $\sup(A_\epsilon \cap \mathbb{Q}) = \sup(A_\varepsilon \cap (\mathbb{R} \setminus \mathbb{Q}))$

126. Let f: $\mathbb{R}\to\mathbb{R}$ be a function satisfying $f(x+y)=f(x)f(y), \forall\ x,y \in \mathbb{R}$ and $\lim_{x\to 0} f(x) = 1$.
Which of the following are necessarily true?

1. f is strictly increasing
2. f is either constant or bounded
3. $f(rx)=f(x)^r$ for every rational $r\in\mathbb{Q}$
4. $f(x) \ge 0,\ \forall\ x\in\mathbb{R}$

127. Evaluate $\lim_{n\to\infty} \sum_{k=0}^{n} \frac{n}{k^2+n^2}$

1. $\frac{\pi}{2}$
2. π
3. $\frac{\pi}{8}$
4. $\frac{\pi}{4}$

128. Let $f(x,y) = \frac{1-\cos(x+y)}{x^2+y^2}$ if $(x,y) \ne (0,0)$, $f(0,0) = \frac{1}{2}$ and

$$g(x,y) = \frac{1-\cos(x+y)}{(x+y)^2} \quad if \quad x+y \ne 0$$

$$g(x,y) = \frac{1}{2} \quad if \quad x+y = 0$$

Then

1. f is continuous at (0,0)
2. f is continuous everywhere except at (0,0)
3. g is continuous at (0,0)
4. g is continuous everywhere

129. Let f: $\mathbb{R}^4\to\mathbb{R}$ be defined by $f(x) = x^t Ax$, where A is a 4×4 matrix with real entries and x^t denotes the transpose of x. The gradient of f at a point x_0 necessarily is

1. $2Ax_0$
2. $Ax_0 + A^t x_a$
3. $2A^t x_0$
4. Ax_0

130. Let f: $\mathbb{R}^n\to\mathbb{R}^n$ be a continuously differentiable map satisfying $\|f(x) - f(y)\| \ge \|x-y\|$, for all $x,y\in \mathbb{R}^n$. Then

1. f is onto
2. $f(\mathbb{R}^n)$ is a closed subset of $\mathbb{R}^n$
3. $f(\mathbb{R}^n)$ is an open subset of $\mathbb{R}^n$
4. $f(0) = 0$

131. Let f : $[-1,1] \to\mathbb{R}$ be a function given by $f(x) = \begin{cases} x^2\cos\left(\frac{1}{x}\right) & if \quad x \ne 0 \\ 0 & if \quad x = 0 \end{cases}$. Then

1. f is of bounded variation on [-1,1]
2. f ′ is of bounded variation on [-1,1]
3. $|f'(x)| \le 1\ \forall\ x\in[-1,1]$
4. $|f'(x)| \le 3\ \forall x\in[-1,1]$

132. For a set X, let P(X) be the set of all subsets of X and let Ω(X) be the set of all functions f:X→ {0,1}. Then

1. If X is finite then P(X) is finite
2. If X and Y are finite sets and if there is a 1-1 correspondence between P(X) and P(Y), then there is a 1-1 correspondence between X and Y
3. There is no 1-1 correspondence between X and P(X)
4. There is a 1-1 correspondence between Ω(X) and P(X)

133. Let d and d′ be metrics on a non-empty set X. Then which of the following are metrices on X?

1. $\rho_1(x,y)=d(x,y)+d'(x,y)$ for all x, y ∈ X
2. $\rho_2(x,y)=d(x,y)d'(x,y)$ for all x, y ∈ X
3. $\rho_3(x,y)=\max\{d(x,y),d'(x,y)\}$ for all x, y ∈ X
4. $\rho_4(x,y)=\min\{d(x,y),d'(x,y)\}$ for all x, y ∈ X

JUNE – 2018

PART – B

134. Given that there are real constants a,b,c,d such that the identity $\lambda x^2+2xy+y^2=(ax+by)^2+(cx+dy)^2$ holds for all x,y ∈ ℝ. This implies

1. $\lambda=-5$ 2. $\lambda\geq 1$ 3. $0<\lambda<1$ 4. there is no such $\lambda\in\mathbb{R}$

135. Given {a_n}, {b_n} two monotone sequences of real numbers and that $\sum a_n b_n$ is convergent, which of the following is true?

1. $\sum a_n$ is convergent and $\sum b_n$ is convergent
2. At least one of $\sum a_n$, $\sum b_n$ is convergent
3. {a_n} is bounded and {b_n} is bounded
4. At least one of {a_n} , {b_n} is bounded

136. Let $S=\{(x,y)\mid x^2+y^2=\frac{1}{n^2}$, where n ∈ ℕ and either x ∈ ℚ or y ∈ ℚ}. Here ℚ is the set of rational numbers and ℕ is the set of positive integers. Which of the following is true?

1. S is a finite non empty set
2. S is countable
3. S is uncountable
4. S is empty

137. Define the sequence {a_n} as follows:

$a_1 = 1$ and for $n \geq 1$, $a_{n+1}=(-1)^n\left(\frac{1}{2}\right)\left(|a_n|+\frac{2}{|a_n|}\right)$. Which of the following is true?

1. $\limsup a_n=\sqrt{2}$
2. $\liminf a_n=-\infty$
3. $\lim a_n=\sqrt{2}$
4. $\sup a_n=\sqrt{2}$

138. If {x_n} is a convergent sequence in ℝ and {y_n} is a bounded sequence in ℝ, then we can conclude that

1. {$x_n + y_n$} is convergent
2. {$x_n + y_n$} is bounded
3. {$x_n + y_n$} has no convergent subsequence
4. {$x_n + y_n$} has no bounded subsequence

139. The difference $\log(2)-\sum_{n=1}^{100}\frac{1}{2^n\cdot n}$ is

1. less than 0
2. greater than 1
3. less than $\frac{1}{2^{100}\cdot 101}$
4. greater than $\frac{1}{2^{100}\cdot 101}$

140. Let $f(x,y)=\log(\cos^2(e^{x^2}))+\sin(x+y)$. Then $\frac{\partial}{\partial y}\frac{\partial}{\partial x}f(x,y)$ is

1. $\frac{\cos(e^{x^2})-1}{1+\sin^2(e^{x^2})}-\cos(x+y)$
2. 0
3. $-\sin(x+y)$
4. $\cos(x+y)$

141. Let $f(x)=x^5-5x+2$. Then

1. f has no real root
2. f has exactly one real root
3. f has exactly three real roots
4. all roots of f are real

142. Consider the space $S=\{(\alpha,\beta)\,|\,\alpha,\beta\in\mathbb{Q}\}\subset\mathbb{R}^2$, where $\mathbb{Q}$ is the set of rational numbers. Then

1. S is connected in $\mathbb{R}^2$
2. S^c is connected in $\mathbb{R}^2$
3. S is closed in $\mathbb{R}^2$
4. S^c is closed in $\mathbb{R}^2$

PART - C

143. For each $\alpha\in\mathbb{R}$, let $S_\alpha=\{(x,y,z)\in\mathbb{R}^3\,|\,x^2+y^2+z^2=\alpha^2\}$. Let $E=\bigcup_{\alpha\in\mathbb{R}\setminus\mathbb{Q}} S_\alpha$. Which of the following are true?

1. The Lebesgue measure of E is infinite
2. E contains a non-empty open set
3. E is path connected
4. Every open set containing E^C has infinite Legesgue measure

144. Which of the following sets are uncountable?

1. The set of all functions from $\mathbb{R}$ to {0, 1}
2. The set of all functions from $\mathbb{N}$ to {0, 1}
3. The set of all finite subsets of $\mathbb{N}$
4. The set of all subsets of $\mathbb{N}$

145. Let $A=\left\{t\sin\left(\frac{1}{t}\right)\middle|\, t\in\left(0,\frac{2}{\pi}\right)\right\}$. Which of the following statements are true?

1. $\sup(A)<\frac{2}{\pi}+\frac{1}{n\pi}$ for all $n\geq 1$
2. $\inf(A)>\frac{-2}{3\pi}-\frac{1}{n\pi}$ for all $n\geq 1$
3. sup (A) = 1
4. inf (A) = -1

146. Let $C_C(\mathbb{R})=\{f:\mathbb{R}\to\mathbb{R}\mid f$ is continuous and there exists a compact set K such that f(x) = 0 for all $x\in K^C\}$. Let $g(x)=e^{-x^2}$ for all $x\in\mathbb{R}$. Which of the following statements are true?

1. There exists a sequence $\{f_n\}$ in $C_C(\mathbb{R})$ such that $f_n\to g$ uniformly

2. There exists a sequence $\{f_n\}$ in $C_C(\mathbb{R})$ such that $f_n \to g$ pointwise
3. If a sequence in $C_C(\mathbb{R})$ converges pointwise to g then it must converge uniformly to g
4. There does not exist any sequence in $C_C(\mathbb{R})$ converging pointwise to g

147. Given that $a(n) = \frac{1}{10^{100}} 2^n$, $b(n) = 10^{100} \log(n)$, $c(n) = \frac{1}{10^{10} n^2}$, which of the following statements are true?

1. a(n) > c(n) for all sufficiently large n
2. b(n) > c(n) for all sufficiently large n
3. b(n) > n for all sufficiently large n
4. a(n) > b(n) for all sufficiently large n

148. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \frac{a}{1+bx^2}$, a, b $\in \mathbb{R}$, b $\geq$ 0. Which of the following are true?

1. f is uniformly continuous on compact intervals of $\mathbb{R}$ for all values of a and b
2. f is uniformly continuous on $\mathbb{R}$ and is bounded for all values of a and b
3. f is uniformly continuous on $\mathbb{R}$ only if b=0
4. f is uniformly continuous on $\mathbb{R}$ and unbounded if a $\neq$ 0, b $\neq$ 0

149. Let $\alpha = \int_0^\infty \frac{1}{1+t^2} dt.$ Which of the following are true?

1. $\frac{d\alpha}{dt} = \frac{1}{1+t^2}$
2. α is a rational number
3. log (α) = 1
4. sin (α) = 1

150. Which of the following functions are of bounded variation?

1. $x^2 + x + 1$ for x $\in$ (-1, 1)
2. $\tan\left(\frac{\pi x}{2}\right)$ for x $\in$ (-1, 1)
3. $\sin\left(\frac{x}{2}\right)$ for x $\in$ ($-\pi$, π)
4. $\sqrt{1-x^2}$ for x $\in$ (-1, 1)

151. For any y $\in \mathbb{R}$, let [y] denote the greatest integer less than or equal to y. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = x^{[y]}$. Then

1. f is continuous on $\mathbb{R}^2$
2. for every y $\in \mathbb{R}$, $x \mapsto f(x, y)$ is continuous on $\mathbb{R}\setminus\{0\}$
3. for every x $\in \mathbb{R}$, $y \mapsto f(x, y)$ is continuous on $\mathbb{R}$
4. f is continuous at no point of $\mathbb{R}^2$

152. Let f(x) $\in \mathbb{Z}[x]$ be a monic polynomial. Then the roots of f

1. can belong to $\mathbb{Z}$
2. always belong to $(\mathbb{R}\setminus\mathbb{Q}) \cup \mathbb{Z}$
3. always belong to $(\mathbb{C}\setminus\mathbb{Q}) \cup \mathbb{Z}$
4. can belong to $(\mathbb{Q}\setminus\mathbb{Z})$

DECEMBER-2018

PART – B

153. Consider the function $\tan x$ on the set $S = \left\{x \in \mathbb{R}: x \geq 0,\ x \neq k\pi + \frac{\pi}{2} \text{ for any } k \in \mathbb{N} \cup \{0\}\right\}$. We say that it has a fixed point in S if $\exists\ x \in S$ such that $\tan x$ = x. Then

1. There is a unique fixed point.
2. There is no fixed point.
3. There are infinitely many fixed points.
4. There are more than one but finitely many fixed points.

154. Define $f(x) = \frac{1}{\sqrt{x}}$ for x > 0. Then f is uniformly continuous

1. on $(0,\infty)$.
2. on $[r,\infty)$ for any r>0.
3. on (0,r] for any r>0.
4. only on intervals of the form [a,b] for $0 < a < b < \infty$.

155. Consider the map f: $\mathbb{Q} \rightarrow \mathbb{R}$ defined by

(i) f(0) = 0

(ii) $f(r) = \frac{p}{10^q}$, where $r = \frac{p}{q}$ with $p \in \mathbb{Z}$, $q \in \mathbb{N}$ and gcd (p,q)=1. Then the map f is

1. one-to-one and onto
2. not one-to-one , but onto
3. onto but not one-to-one
4. neither one-to-one nor onto

156. Let x be a real number such that |x| < 1. Which of the following is FALSE?

1. If $x \in \mathbb{Q}$, then $\sum_{m\geq 0} x^m \in \mathbb{Q}$
2. If $\sum_{m\geq 0} x^m \in \mathbb{Q}$, then $x \in \mathbb{Q}$
3. If $x \notin \mathbb{Q}$, then $\sum_{m\geq 0} m x^{m-1} \notin \mathbb{Q}$
4. $\sum_{m\geq 1} \frac{x^m}{m}$ converges in $\mathbb{R}$

157. Suppose that $\{x_n\}$ is a sequence of real numbers satisfying the following. For every $\varepsilon>0$, there exists n_0 such that $|x_{n+1}-x_n| < \varepsilon\ \forall\ n \geq n_0$. The sequence $\{x_n\}$ is

1. bounded but not necessarily Cauchy.
2. Cauchy but not necessarily bounded.
3. Convergent.
4. not necessarily bounded.

158. Let $A(n) = \int_n^{n+1} \frac{1}{x^3}dx$ for n ≥ 1. For $c \in \mathbb{R}$, let $\lim_{n\to\infty} n^c A(n) = L$. Then

1. L=0 if c > 3
2. L=1 if c=3
3. L=2 if c=3
4. $L=\infty$ if 0< c<3

PART - C

159. Let f : $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a function given by f(x, y) = $(x^3 + 3xy^2 - 15x - 12y,\ x + y)$. Let S = {(x, y) $\in \mathbb{R}^2$: f is locally invertible at (x, y)}. Then

1. S = $\mathbb{R}^2 \backslash \{(0, 0)\}$
2. S is open in $\mathbb{R}^2$
3. S is dense in $\mathbb{R}^2$
4. $\mathbb{R}^2 \backslash S$ is countable

160. Let X = ℕ, the set of positive integers. Consider the metrices d_1, d_2 on X given by $d_1(m, n) = |m - n|$, m, n∈X, $d_2(m,n)=\left|\frac{1}{m}-\frac{1}{n}\right|, m, n \in X$. Let X_1, X_2 denote the metric spaces (X, d_1), (X, d_2) respectively. Then

1. X_1 is complete
2. X_2 is complete
3. X_1 is totally bounded
4. X_2 is totally bounded

161. Let $\{u_n\}_{n\geq 1}$ be a sequence of real numbers satisfying the following conditions:

(1) $(-1)^n u_n \geq 0$, for all $n\geq 1$

(2) $|u_{n+1}| < \frac{|u_n|}{2}$, for all $n \geq 13$

Which of the following statements are necessarily true?

1. $\sum_{n\geq 1} u_n$ does not converge in ℝ.
2. $\sum_{n\geq 13} u_n$ converges to zero.
3. $\sum_{n\geq 13} u_n$ converges to a non-zero real number.
4. If $|u_{n-1}| < \frac{|u_n|}{2}$, for all $2 \leq n \leq 13$, then $\sum_{n\geq 1} u_n$ is a negative real number.

162. Let S be an infinite set. Which of the following statements are true?

1. If there is an injection from S to ℕ, then S is countable
2. If there is a surjection from S to ℕ, then S is countable
3. If there is an injection from ℕ to S, then S is countable
4. If there is a surjection from ℕ to S, then S is countable

163. Let p_n denote the n-th prime number, when we enumerate the prime numbers in the increasing order. For example, $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, and so on. Let $S = \{s_n = p_{n+1} - p_n | n \in \mathbb{N}, n \geq 1\}$. Then which of the following are correct?

1. $\sup S = \infty$
2. $\limsup_{n\to\infty} s_n = \infty$
3. $\inf S < \infty$ and $\inf S = 1$
4. $\liminf_{n\to\infty} s_n \geq 2$

164. For n ≥ 1, consider the sequence of functions $f_n(x)=\frac{1}{2nx+1}, g_n(x)=\frac{x}{2nx+1}$ on the open interval (0, 1). Consider the statements:

(I) The sequence $\{f_n\}$ converges uniformly on (0, 1)

(II) The sequence $\{g_n\}$ converges uniformly on (0, 1). Then,

1. (I) is true
2. (I) is false
3. (I) is false and (II) is true
4. Both (I) and (II) are true

165. Suppose that $\{f_n\}$ is a sequence of continuous real valued functions on [0, 1] satisfying the following:

(A) $\forall x \in \mathbb{R}$, $\{f_n(x)\}$ is a decreasing sequence

(B) the sequence $\{f_n\}$ converges uniformly to 0.

Let $g_n(x)=\sum_{k=1}^{n}(-1)^k f_k(x)\ \forall\ x \in \mathbb{R}$. Then

1. $\{g_n\}$ is Cauchy with respect to the sup norm
2. $\{g_n\}$ is uniformly convergent
3. $\{g_n\}$ need not converge pointwise
4. $\exists$ M > 0 such that $|g_n(x)| \leq M$, $\forall n\in\mathbb{N}$, $\forall x\in\mathbb{R}$

166. Given $f:\left[\frac{1}{2},2\right]\to\mathbb{R}$, a strictly increasing function, we put g(x)=f(x)+f(1/x), $x\in[1, 2]$. Consider a partition P of [1, 2] and let U (P, g) and L (P, g) denote the upper Riemann sum and lower Riemann sum of g. Then

1. for a suitable f we can have U (P, g) = L (P, g)
2. for a suitable f we can have U(P, g) $\neq$L (P, g)
3. U (P, g) $\geq$ L (P, g) for all choices of f
4. U (P, g) < L (P, g) for all choices of f

167. Let f be a real valued continuously differentiable function of (0, 1). Set g = f ′ + if, where i^2=-1 and f ′ is the derivative of f. Let a, b$\in$(0,1) be two consecutive zeros of f. Which of the following statements are necessarily true?

1. If g(a) > 0, then g crosses the real line from upper half plane to lower half plane at a
2. If g(a)>0, then g crosses the real line from lower half plane to upper half plane at a
3. if g(a) g(b)$\neq$0, then g(a), g(b) have the same sign
4. If g(a) g(b)$\neq$0, then g(a), g(b) have opposite signs

JUNE -2019

PART – B

168. Which of the following sets is uncountable?

1. $\left\{x\in\mathbb{R} \mid \log(x) = \frac{p}{q} \text{ for some } p, q\in\mathbb{N}\right\}$
2. $\{x\in\mathbb{R} \mid (\cos(x))^n + (\sin(x))^n = 1 \text{ for some } n\in\mathbb{N}\}$
3. $\left\{x\in\mathbb{R} \mid x = \log\left(\frac{p}{q}\right) \text{ for some } p, q\in\mathbb{N}\right\}$
4. $\left\{x\in\mathbb{R} \mid \cos(x) = \frac{p}{q} \text{ for some } p, q\in\mathbb{N}\right\}$

169. Consider a sequence $\{a_n\}$, $a_n=(-1)^n\left(\frac{1}{2}-\frac{1}{n}\right)$. Let $b_n=\sum_{k=1}^{n}a_k$ $\forall$ $n\in\mathbb{N}$.Then which of the following is true?

1. $\lim_{n\to\infty}b_n = 0$
2. $\limsup_{n\to\infty}b_n > 1/2$
3. $\liminf_{n\to\infty}b_n < -1/2$
4. $0 \leq \liminf_{n\to\infty}b_n \leq \limsup_{n\to\infty}b_n \leq 1/2$

170. Which of the following is true?

1. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ does not converge
2. $\sum_{n=1}^{\infty}\frac{1}{n}$ converges
3. $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{(m+n)^2}$ converges
4. $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{(m+n)^2}$ diverges

171. For $n\in\mathbb{N}$, which of the following is true?

1. $\sqrt{n+1}-\sqrt{n}>\frac{1}{\sqrt{n}}$ for all, except possibly finitely many n

2. $\sqrt{n+1}-\sqrt{n}<\frac{1}{\sqrt{n}}$ for all, except possibly finitely many n
3. $\sqrt{n+1}-\sqrt{n}>1$ for all, except possibly finitely many n
4. $\sqrt{n+1}-\sqrt{n}>2$ for all, except possibly finitely many n

172. Let f : ℝ→ℝ be a continuous and one-one function. Then which of the following is true?
1. f is onto
2. f is either strictly decreasing or strictly increasing
3. there exists x ∈ℝ such that f(x) = 1
4. f is unbounded

173. Let $g_n(x)=\frac{nx}{1+n^2x^2}, x\in[0,\infty)$. Which of the following is true as n →∞?
1. $g_n \to 0$ pointwise but not uniformly
2. $g_n \to 0$ uniformly
3. $g_n(x) \to x \quad \forall x \in [0, \infty)$
4. $g_n(x) \to \frac{x}{1+x^2}\ \forall x \in [0, \infty)$

PART - C

174. Let $\{a_n\}_{n\geq 0}$ be a sequence of positive real numbers. Then, for $K=\limsup_{n\to\infty}|a_n|^{\frac{1}{n}}$, which of the following are true?
1. if K = ∞, then $\sum_{n=0}^{\infty}a_nr^n$ is convergent for every r > 0
2. if K = ∞, then $\sum_{n=0}^{\infty}a_nr^n$ is not convergent for any r > 0
3. if K = 0, then $\sum_{n=0}^{\infty}a_nr^n$ is convergent for every r > 0
4. if K = 0, then $\sum_{n=0}^{\infty}a_nr^n$ is not convergent for any r > 0

175. For α∈ℝ, let [α] denote the greatest integer smaller than or equal to α. Define d : ℝ×ℝ→ [0, ∞) by d (x, y) = [|x – y|], x, y ∈ℝ. Then which of the following are true?
1. d(x, y) = 0 if and only if x = y, x, y ∈ℝ
2. d(x, y) = d(y, x), x, y ∈ℝ
3. d(x, y) ≤ d(x, z) + d(z, y), x, y, z ∈ℝ
4. d is not a metric on ℝ

176. Consider a function f : ℝ→ℝ. Then which of the following are true?
1. f is not one-one if the graph of f intersects some line parallel to X-axis in at least two points
2. f is one-one if the graph of f intersects any line parallel to the X-axis in at most one point
3. f is surjective if the graph of f intersects every line parallel to X-axis
4. f is not surjective if the graph of f does not intersect at least one line parallel to X-axis

177. Let $f(x)=\int_1^{\infty}\frac{\cos t}{x^2+t^2}dt.$ Then which of the following are true?
1. f is bounded on ℝ
2. f is continuous on ℝ
3. f is not defined everywhere on ℝ
4. f is not continuous on ℝ

178. Suppose that $\{x_n\}$ is a sequence of positive reals. Let $y_n=\frac{x_n}{1+x_n}$. Then which of the following are true?
1. $\{x_n\}$ is convergent if $\{y_n\}$ is convergent
2. $\{y_n\}$ is convergent if $\{x_n\}$ is convergent
3. $\{y_n\}$ is bounded if $\{x_n\}$ is bounded
4. $\{x_n\}$ is bounded if $\{y_n\}$ is bounded

179. Let $f(x) = \begin{cases} x\sin(1/x), & for\, x \in (0,1] \\ 0, & for\, x = 0 \end{cases}$ and g(x) = x f(x) for $0 \le x \le 1$. Then which of the following are true?

1. f is of bounded variation
2. f is not of bounded variation
3. g is of bounded variation
4. g is not of bounded variation

180. Let a < c < b, f : (a, b) $\to \mathbb{R}$ be continuous. Assume that f is differentiable at every point of (a, b) \ {c} and f′ has a limit at c. Then which of the following are true?

1. f is differentiable at c
2. f need not be differentiable at c
3. f is differentiable at c and $\lim_{x\to c} f'(x) = f'(c)$
4. f is differentiable at c but f′(c) is not necessarily $\lim_{x\to c} f'(x)$

181. Let F: $\mathbb{R}\to\mathbb{R}$ be a non-decreasing function. Which of the following can be the set of discontinuities of F

1. $\mathbb{Z}$
2. $\mathbb{N}$
3. $\mathbb{Q}$
4. $\mathbb{R}\setminus\mathbb{Q}$

182. Let f : $\mathbb{R}^3\to\mathbb{R}^3$ be given by $f(x_1, x_2, x_3) = (e^{x_2}\cos x_1, e^{x_2}\sin x_1, 2x_1 - \cos x_3)$. Consider

E = $\{(x_1, x_2, x_3) \in \mathbb{R}^3$: there exists an open subset U around (x_1, x_2, x_3) such that $f|_U$ is an open map$\}$. Then which of the following are true?

1. $E = \mathbb{R}^3$
2. E is countable
3. E is not countable but not $\mathbb{R}^3$
4. $\left\{\left(x_1, x_2, \frac{\pi}{2}\right) \in \mathbb{R}^3 : x_1, x_2 \in \mathbb{R}\right\}$ is a proper subset of E

183. Let X be a countable set. Then which of the following are true?

1. There exists a metric d on X such that (X, d) is complete
2. There exists a metric d on X such that (X, d) is not complete
3. There exists a metric d on X such that (X, d) is compact
4. There exists a metric d on X such that (X, d) is not compact

DECEMBER-2019

PART – B

184. Let $\le$ be the usual order on the field $\mathbb{R}$ of real numbers. Define an order $\le$ on $\mathbb{R}^2$ by (a, b) $\le$ (c, d) if (a < c) or (a = c and b $\le$ d). Consider the subset

E = $\left\{\left(\frac{1}{n}, 1-\frac{1}{n}\right) \in \mathbb{R}^2 : n \in \mathbb{N}\right\}$. With respect to $\le$ which of the following statements is true?

1. inf(E) = (0, 1) and sup (E) = (1, 0)
2. inf(E) does not exist but sup(E) = (1, 0)
3. inf(E) = (0, 1) but sup(E) does not exist
4. Both inf(E) and sup(E) do not exist.

185. Which of the following sets is countable?

1. The set of all functions from $\mathbb{Q}$ to $\mathbb{Q}$
2. The set of all functions from $\mathbb{Q}$ to {0, 1}
3. The set of all functions from $\mathbb{Q}$ to {0, 1} which vanish outside a finite set
4. The set of all subsets of $\mathbb{N}$

186. Let E = $\left\{\frac{1}{n} \mid n \in \mathbb{N}\right\}$. For each m $\in\mathbb{N}$ define f_m : E $\to\mathbb{R}$ by $f_m(x) = \begin{cases} \cos(mx)\, if\;\; x \geq \frac{1}{m} \\ 0\, if\;\; \frac{1}{m+10} < x < \frac{1}{m} \\ x\, if\;\; x \leq \frac{1}{m+10} \end{cases}$

Then which of the following statements is true?
1. No subsequence of $(f_m)_{m\geq1}$ converges at every point of E
2. Every subsequence of $(f_m)_{m\geq1}$ converges at every point of E
3. There exist infinitely many subsequences of $(f_m)_{m\geq1}$ which converge at every point of E
4. There exists a subsequence of $(f_m)_{m\geq1}$ which converges to 0 at every point of E

187. Let $(x_n)_{n\geq1}$ be a sequence of non –negative real numbers. Then which of the following is true?
1. liminf $x_n = 0 \Rightarrow \lim x_n^2 = 0$
2. limsup $x_n = 0 \Rightarrow \lim x_n^2 = 0$
3. liminf $x_n = 0 \Rightarrow (x_n)_{n\geq1}$ is bounded
4. liminf $x_n^2 > 4 \Rightarrow$ limsup $x_n > 4$

188. Let X $\subset\mathbb{R}$ be an infinite countable bounded subset of $\mathbb{R}$. Which of the following statements is true?
1. X cannot be compact
2. X contains an interior point
3. X may be closed
4. closure of X is countable

189. What is the sum of the following series?

$$\left(\frac{1}{2.3}+\frac{1}{2^2.3}\right)+\left(\frac{1}{2^2.3^2}+\frac{1}{2^3.3^2}\right)+\ldots+\left(\frac{1}{2^a.3^a}+\frac{1}{2^{a+1}.3^a}\right)+\ldots$$

1. $\frac{3}{8}$
2. $\frac{3}{10}$
3. $\frac{3}{14}$
4. $\frac{3}{16}$

PART – C

190. Let $L^2([-\pi, \pi])$ be the metric space of Lebesgue square integrable functions on $[-\pi, \pi]$ with a metric d given by $d(f,g) = \left[\int_{-\pi}^{\pi}(f(x)-g(x))^2\,dx\right]^{1/2}$ for f, g $\in L^2([-\pi, \pi])$. Consider the subset

S = $\{\sin(2^n x) : n \in\mathbb{N}\}$ of $L^2([-\pi, \pi])$.Which of the following statements are true?
1. S is bounded
2. S is closed
3. S is compact
4. S is non-compact

191. Let f :$[0, 1]^2\to\mathbb{R}$ be a function defined by $f(x,y) = \frac{xy}{x^2+y^2}$ if either x≠0 or y≠0 = 0 if x = y = 0.

Then which of the following statements are true?
1. f is continuous at (0, 0)
2. f is a bounded function
3. $\int_0^1\int_0^1 f(x,y)\,dxdy$ exists
4. f is continuous at (1, 0)

192. Let p(x) be a polynomial function in one variable of odd degree and g be a continuous function from $\mathbb{R}$ to $\mathbb{R}$. Then which of the following statements are true?

1. $\exists$ a point $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$
2. If g is a polynomial function then there exists $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$
3. If g is a bounded function there exists $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$
4. There is a unique point $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$

193. Let f(x) be a real polynomial of degree 4. Suppose f(-1)=0, f(0)=0, f(1)=1 and $f^{(1)}(0) = 0$, where $f^{(k)}(a)$ is the value of k^{th} derivative of f(x) at x=a. Which of the following statements are true?

1. There exists $a \in (-1,1)$ such that $f^{(3)}(a) \geq 3$
2. $f^{(3)}(a) \geq 3$ for all $a \in (-1, 1)$
3. $0 < f^{(3)}(0) \leq 2$
4. $f^{(3)}(0) \geq 3$

194. Let (X, d) be a compact metric space. Let $T : X \to X$ be a continuous function satisfying $\inf_{n\in N} d(T^n(x), T^n(y)) \neq 0$ for every $x, y \in X$ with $x \neq y$. Then which of the following statements are true?

1. T is a one-one function
2. T is not a one-one function
3. Image of T is closed in X
4. If X is finite, then T is onto

195. For each natural number $n \geq 1$, let $a_n = \dfrac{n}{10^{[\log_{10} n]}}$, where [x] = smallest integer greater than or equal to x. Which of the following statements are true?

1. $\lim_{n\to\infty} \inf a_n = 0$
2. $\lim_{n\to\infty} \inf a_n$ does not exist
3. $\lim_{n\to\infty} \inf a_n = 0.15$
4. $\lim_{n\to\infty} \sup a_n = 1$

196. Let $U \subseteq \mathbb{R}^n$ be an open subset of $\mathbb{R}^n$ and $f : U \to \mathbb{R}^n$ be a C^∞ - function. Suppose that for every $x \in U$, the derivative at x, df_x, is non-singular. Then which of the following statements are true?

1. If $V \subset U$ is open then f(V) is open in $\mathbb{R}^n$
2. $f : U \to f(U)$ is a homomorphism
3. f is one-one
4. If $V \subset U$ is closed, then f(V) is closed in $\mathbb{R}^n$.

197. Let n be a fixed natural number. Then the series $\sum_{m\geq n} \dfrac{(-1)^m}{m}$ is

1. Absolutely convergent
2. Divergent
3. Absolutely convergent if n > 100
4. Convergent

198. Let $\{a_n\}_{n\geq 1}$ be a bounded sequence of real numbers. Then

1. Every subsequence of $\{a_n\}_{n\geq 1}$ is convergent
2. There is exactly one subsequence of $\{a_n\}_{n\geq 1}$ which is convergent
3. There are infinitely many subsequences of $\{a_n\}_{n\geq 1}$ which are convergent
4. There is a subsequence of $\{a_n\}_{n\geq 1}$ which is convergent

199. Let $N \geq 5$ be an integer. Then which of the following statements are true?

1. $\sum_{n=1}^{N} \dfrac{1}{n} \leq 1 + \log N$
2. $\sum_{n=1}^{N} \dfrac{1}{n} < 1 + \log N$
3. $\sum_{n=1}^{N} \dfrac{1}{n} \leq \log N$
4. $\sum_{n=1}^{N} \dfrac{1}{n} \geq \log N$

200. Let f : [0, 1] $\to\mathbb{R}$ be a monotonic function with $f\left(\frac{1}{4}\right)f\left(\frac{3}{4}\right)<0.$

Suppose sup{x $\in$ [0, 1] : f(x) < 0} = α. Which of the following statements are correct?
1. f(α) < 0
2. If f is increasing, then f(α) $\leq$0
3. If f is continuous and $\frac{1}{4}<\alpha<\frac{3}{4}$, then f($\alpha$) = 0
4. If f is decreasing, then f(α) < 0

201. Which of the following statements are true?
1. There exist three mutually disjoint subsets of $\mathbb{R}$, each of which is countable and dense in $\mathbb{R}$
2. For each n $\in\mathbb{N}$, there exist n mutually disjoint subsets of $\mathbb{R}$ each of which is countable and dense in $\mathbb{R}$
3. There exists countably infinite number of mutually disjoint subsets of $\mathbb{R}$, each of which is countable and dense in $\mathbb{R}$
4. There exist uncountable number of mutually disjoint subsets of $\mathbb{R}$, each of which is countable and dense in $\mathbb{R}$

JUNE-2020

PART – B

202. Let {E_n} be a sequence of subsets of $\mathbb{R}$.
Define

$$\lim_n \sup E_n = \bigcap_{k=1}^{\infty}\bigcup_{n=k}^{\infty}E_n$$

$$\lim_n \inf E_n = \bigcup_{k=1}^{\infty}\bigcap_{n=k}^{\infty}E_n$$

Which of the following statements is true?
1. $\lim_n \sup E_n = \lim_n \inf E_n$
2. $\lim_n \sup E_n = \{x : x\in E_n \ for\, some\ n\}$
3. $\lim_n \inf E_n = \{x : x\in E_n \ for\, all\, but\ finitely\, many\, n\}$
4. $\lim_n \inf E_n = \{x : x\in E_n \ for\, \text{infinitely} \ many\, n\}$

203. f : $\mathbb{N}\to\mathbb{N}$ be a bounded function. Which of the following statements is NOT true?
1. $\lim_{n\to\infty} \sup f(n)\in\mathbb{N}$
2. $\lim_{n\to\infty} \inf f(n)\in\mathbb{N}$
3. $\lim_{n\to\infty} \inf (f(n)+n)\in\mathbb{N}$
4. $\lim_{n\to\infty} \sup (f(n)+n)\notin\mathbb{N}$

204. Which of the following statements is true?
1. There are at most countably many continuous maps from $\mathbb{R}^2$ to $\mathbb{R}$.
2. There are at most finitely many continuous surjective maps from $\mathbb{R}^2$ to $\mathbb{R}$.
3. There are infinitely many continuous injective maps from $\mathbb{R}^2$ to $\mathbb{R}$.

4. There are no continuous bijective maps from $\mathbb{R}^2$ to $\mathbb{R}$.

205. The series $\sum_{n=1}^{\infty} \frac{(-1)^n \sin nx}{n^{\log_e n}}, x \in \mathbb{R}$ converges

1. only for x = 0
2. uniformly only for $x \in [-\pi, \pi]$
3. uniformly only for $x \in \mathbb{R}\backslash\{n\pi : n \in \mathbb{Z}\}$
4. uniformly for all $x \in \mathbb{R}$

206. Given $(a_n)_{n\geq 1}$ a sequence of real numbers, which of the following statements is true?

1. $\sum_{n\geq 1} (-1)^n \frac{a_n}{1+|a_n|}$ converges
2. There is a subsequence $(a_{n_k})_{k\geq 1}$ such that $\sum_{k\geq 1} \frac{a_{n_k}}{1+|a_{n_k}|}$ converges
3. There is a number b such that $\sum_{n\geq 1} \left| b - \frac{a_n}{1+|a_n|} \right| (-1)^n$ converges
4. There is a number b and a subsequence $(a_{n_k})_{k\geq 1}$ such that $\sum_{k\geq 1} \left| b - \frac{a_{n_k}}{1+|a_{n_k}|} \right|$ converges

207. Given f, g are continuous functions on [0, 1] such that f(0) = f(1) = 0; g(0) = g(1) = 1 and f(1/2) > g(1/2). Which of the following statements is true?

1. There is no $t\in[0, 1]$ such that f(t) = g(t)
2. There is exactly one $t\in[0, 1]$ such that f(t) = g(t)
3. There are at least two $t\in[0, 1]$ such that f(t) = g(t)
4. There are always infintely many $t\in[0, 1]$ such that f(t) = g(t)

PART – C

208. Which of the following sets are in bijection with $\mathbb{R}$?

1. Set of all maps from {0, 1} to $\mathbb{N}$
2. Set of all maps from $\mathbb{N}$ to {0, 1}
3. Set of all subsets of $\mathbb{N}$
4. Set of all susets of $\mathbb{R}$

209. Which of the following statements are true?

1. The series $\sum_{n\geq 1} \frac{(-1)^n}{\sqrt{n}}$ is convergent
2. The series $\sum_{n\geq 1} \frac{(-1)^n}{\sqrt{n}+n}$ is absolutely convergent
3. The series $\sum_{n\geq 1} \frac{[1+(-1)^n]\sqrt{n}+\log n}{n^{3/2}}$ is convergent
4. The series $\sum_{n\geq 1} \frac{((-1)^n\sqrt{n}+1)}{n^{3/2}}$ is convergent

210. Let f : $\mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

Define $g(x,y) = \sum_{n=1}^{\infty} \dfrac{f((x-n),(y-n))}{2^n}$

Which of the following statements are true?

1. The function h(y) = g(c, y) is continuous on $\mathbb{R}$ for all c
2. g is continuous from $\mathbb{R}^2$ into $\mathbb{R}$
3. g is not a well-defined function
4. g is continuous on $\mathbb{R}^2 \backslash \{(k, k)\}_{k \in \mathbb{N}}$

211. Consider the series

$A(x) = \sum_{n=0}^{\infty} x^n (1-x)$ and $B(x) = \sum_{n=0}^{\infty} (-1)^n x^n (1-x)$

where x $\in$ [0, 1].

Which of the following statements are true?

1. Both A(x) and B(x) converge pointwise
2. Both A(x) and B(x) converge uniformly
3. A(x) converges uniformly but B(x) does not
4. B(x) converges uniformly but A(x) does not

212. For p $\in \mathbb{R}$, consider the improper integral

$$I_p = \int_0^1 t^p \sin t \, dt.$$

Which of the following statements are true?

1. I_p is convergent for p = -1/2
2. I_p is divergent for p = -3/2
3. I_p is convergent for p = 4/3
4. I_p is divergent for p = -4/3

213. Suppose that $\{f_n\}$ is a sequence of real-valued functions on $\mathbb{R}$. Suppose it converges to a continuous function f uniformly on each closed and bounded subset of $\mathbb{R}$. Which of the following statements are true?

1. The sequence $\{f_n\}$ converges to f uniformly on $\mathbb{R}$
2. The sequence $\{f_n\}$ converges to f pointwise on $\mathbb{R}$
3. For all sufficiently large n, the function f_n is bounded
4. For all sufficiently large n the function f_n is continuous

214. Let f(x) = e^{-x} and g(x) = e^{-x^2}. Which of the following statements are true?

1. Both f and g are uniformly continuous on $\mathbb{R}$
2. f is uniformly continuous on every interval of the form [a, +∞), a $\in \mathbb{R}$
3. g is uniformly continuous on $\mathbb{R}$
4. f(x) g(x) is uniformly continuous on $\mathbb{R}$

215. Define

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2} & for\ (x,y) \neq (0,0) \\ 0 & for\ (x,y) = (0,0) \end{cases}$$

Which of the following statements are true?
1. f is discontinuous at (0, 0)
2. f is continuous at (0, 0)
3. all directional derivatives of f at (0, 0) exist
4. f is not differentiable at (0, 0)

216. Define

$$f(x,y)=\begin{cases}\dfrac{x^2-y^2}{x^2+y^2} & for\ (x,y)\neq(0,0)\\ 0 & for\ (x,y)=(0,0)\end{cases}.$$

Which of the following statements are true?
1. f is continuous at (0, 0)
2. f is bounded in a neighbourhood of (0, 0)
3. f is not bounded in any neighbourhood of (0, 0)
4. f has all directional derivatives at (0, 0)

217. Let p : $\mathbb{R}^2 \to \mathbb{R}$ be defined by

$$p(x,y)=\begin{cases}|x| & if \quad x\neq 0\\ |y| & if \quad x=0\end{cases}$$

Which of the following statements are true?
1. p(x, y) = 0 if and only if x = y = 0
2. p(x, y) $\geq$ 0 for all x, y
3. p(αx, αy) = |α| p(x, y) for all $\alpha \in \mathbb{R}$ and for all x, y
4. $p(x_1 + x_2, y_1 + y_2) \leq p(x_1, y_1) + p(x_2, y_2)$ for all (x_1, y_1), (x_2, y_2)

218. Consider the subset of $\mathbb{R}^2$ defined as follows:

A = {(x, y) $\in \mathbb{R} \times \mathbb{R}$: (x – 1) (x – 2) (y – 3) (y + 4) = 0}

Which of the following statements are true?
1. A is connected
2. A is compact
3. A is closed
4. A is dense

JUNE-2020 (Tamil Nadu)

PART – B

219. Suppose that A, B are two non-empty subsets of $\mathbb{R}$ and C = A $\cap$ B. Which of the following conditions imply that C is empty?
1. A and B are open and C is compact
2. A and B are open and C is closed
3. A and B are both dense in $\mathbb{R}$
4. A is open and B is compact

220. $$\lim_{n\to\infty}\frac{((n+1)(n+2)...(n+n))^{1/n}}{n}$$

1. is equal to $\frac{e}{4}$
2. is equal to $\frac{4}{e}$
3. is equal to e
4. does not exist

221. Let Y = {1, 2, 3, …, 100} and let h : Y $\to$ Y be a strictly increasing function. The total number of functions g : Y $\to$ Y satisfying g(h(j)) = h(g(j)), $\forall$ j $\in$ Y is
1. 0
2. 100!
3. 100^{100}
4. 100^{98}

222. An infinite binary word a is a string $(a_1a_2a_3 \ldots)$, where each $a_n \in \{0, 1\}$. Fix a word $s = (s_1s_2s_3\ldots)$, where $s_n = 1$ if and only if n is prime. Let $S = \{a = (a_1a_2a_3 \ldots) \mid \exists m\in\mathbb{N}$ such that $a_n = s_n, \forall n \geq m\}$. What is the cardinality of S?
1. 1
2. Finite but more than 1
3. Countably infinite
4. Uncountable

223. Let f be a non-constant polynomial of degree k and let $g : \mathbb{R} \to \mathbb{R}$ be a bounded continuous function. Which of the following statements is necessarily true?
1. There always exists $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$
2. There is no $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$
3. There exists $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$ is k is even
4. There exists $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$ is k is odd

224. The sum of the infinite series
$$S = \frac{1}{2} - \frac{1}{3\times 1!} + \frac{1}{4\times 2!} - \frac{1}{5\times 3!} + \ldots$$
is equal to
1. $2-\frac{1}{e}$ 2. $1-\frac{2}{e}$ 3. $\frac{2}{e}-1$ 4. $\frac{1}{e}-2$

225. Suppose $f : [0, 1] \times [0, 1] \to (0, 1) \times (0, 1)$ is a continuous non-constant function. Which of the following statements is NOT true?
1. Image of f is uncountable
2. Image of f is a path connected set
3. Image of f is a compact set
4. Image of f has non-empty interior

PART – C

226. Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. Which of the following functions from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ are injective?
1. $f_1(m, n) = 2^m3^n$
2. $f_2(m, n) = mn + m + n$
3. $f_3(m, n) = m^2 + n^3$
4. $f_4(m, n) = m^2n^3$

227. Let $\{x_n\}$ be a sequence of positive real numbers. Which of the following statements are true?
1. If the two subsequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ converge, then the sequence $\{x_n\}$ converges
2. If $\{(-1)^n x_n\}$ converges, then the sequence $\{x_n\}$ converges
3. If $\lim_{n\to\infty} \frac{x_{n+1}}{x_n}$ exists then $\{(x_n)^{1/n}\}$ is bounded
4. If the sequence $\{x_n\}$ is unbounded then every subsequence is unbounded

228. Let $f(x) = e^x$ for $x \in \mathbb{R}$. Which of the following statements are correct?
1. There is a real C > 0 such that $|f(x) - 1 - x| \leq Cx^2$ for all $x \in \mathbb{R}$
2. There is a real C > 0 such that $|f(x) - 1 - x - \frac{x^2}{2}| \leq C|x|^3$ for all $x \in [-1, 1]$
3. There is a real C > 0 such that $|f(x) - 1 - x - \frac{x^2}{2} - \frac{x^2}{3!}| \leq Cx^4$ for all $x \in \mathbb{R}$

4. There is a real C > 0 such that $\left|f(x) - 1 - x - \frac{x^2}{2} - \frac{x^2}{3!}\right| \le Cx^4$ for all $x \in [-1, 1]$

229. Which of the following functions are uniformly continuous on (0, 1)?

1. $\frac{1}{x}$ 2. $\sin\frac{1}{x}$ 3. $x\sin\frac{1}{x}$ 4. $\frac{\sin x}{x}$

230. Let x, y be real numbers such that $0 < y \le x$ and let n be a positive integer. Which of the following statements are true?

1. $ny^{n-1}(x - y) \le x^n - y^n$
2. $nx^{n-1}(x - y) \le x^n - y^n$
3. $ny^{n-1}(x - y) \ge x^n - y^n$
4. $nx^{n-1}(x - y) \ge x^n - y^n$

231. Consider the identity function f(x) = x on I: = [0, 1]. Let P_n be the partition that divides I into n equal parts. If $U(f, P_n)$ and $L(f, P_n)$ are the upper and lower Riemann sums, respectively, and $A_n = U(f, P_n) - L(f, P_n)$ then

1. $\lim_{n\to\infty} nA_n = 0$
2. $\sum_{n=1}^{\infty} A_n$ is convergennt
3. A_n is strictly monotonically decreasing
4. $\sum_{n=1}^{\infty} A_n A_{n+1} = 1$

232. For any two non-negative integers n, k defined $f_{n,k}(x)$ on [0, 1] by

$$f_{n,k}(x) = \begin{cases} x^n \sin\left(\frac{\pi}{2x}\right) - x^k & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

In which of the following cases is the function $f_{n,k'}$ a function of bounded variation?

1. for all $n \ge 1$ and for all $k \ge 0$
2. for all $n \ge 1$ and $k = 0$
3. for all $n \ge 0$ and for all $k \ge 2$
4. for all $n \ge 2$ and for all $k \ge 0$

233. Which of the following functions f admit an inverse in an open neighbourhood of the point f(p)?

1. For p = (1, 0) and $f(x, y) = (x^3 \exp y + y - 2x, 2xy + 2x)$
2. For p = (1, π) and $f(r, \theta) = (r\cos\theta, r\sin\theta)$
3. For p = 0 and $f(x) = \begin{cases} x + 2x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$
4. For p = 0 and $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$

234. Let $f : [0, 1] \to (0, 1)$ be a function. Which of the following statements are FALSE?

1. If f is onto, then f is continuous
2. If f is continuous, then f is not onto
3. If f is one-to-one, then f is continuous
4. If f is continuous, then f is not one-to-one

235. Let $\mathbb{N} = \{1, 2 \ldots\}$ denote the set of positive integers. For $n \in \mathbb{N}$, let

$A_n = \{(x, y, z) \in \mathbb{N}^3 : x^n + y^n = z^n \text{ and } z < n\}$.

Let F(n) be the cardinality of the set A_n. Which of the following statements are true?

1. F(n) is always finite for $n \ge 3$
2. $F(2) = \infty$
3. F(n) = 0 for all n
4. F(n) is non-zero for some n > 2

236. Let S be the set of all continuous functions f : [0, 1] $\to$ [0, ∞) that satisfy

$$\int_0^1 x^2 f(x)\,dx = \frac{1}{2}\int_0^1 x f^2(x)\,dx + \frac{1}{8}$$

Which of the following statements are true?

1. S is an empty set
2. S has atmost one element
3. S has at least one element
4. S has more than two elements

JUNE-2021

PART – B

237. Let S = {1, 2, …, 100} and let A = {1, 2, …, 10} and B = {41, 42, …, 50}. What is the total number of subsets of S, which have non-empty intersection with both A and B?

1. $\dfrac{2^{100}}{2^{20}}$
2. $\dfrac{100!}{10!10!}$
3. $2^{80}(2^{10}-1)^2$
4. $2^{100} - 2(2^{10})$

238. Consider the sequence $\{a_n\}_{n\geq 1}$, where $a_n = 3 + 5\left(-\frac{1}{2}\right)^n + (-1)^n\left(\frac{1}{4} + (-1)^n\frac{2}{n}\right)$

Then the interval $\left(\liminf_{n\to\infty} a_n, \limsup_{n\to\infty} a_n\right)$ is given by

1. (-2, 8)
2. $\left(\frac{11}{4}, \frac{13}{4}\right)$
3. (3, 5)
4. $\left(\frac{1}{4}, \frac{7}{4}\right)$

239. Let f, g: $\mathbb{R} \mapsto \mathbb{R}$ be given by and f(x) = x^2 and g(x) = sinx

Which of the following functions is uniformly continuous on $\mathbb{R}$?

1. h(x) = g(f(x))
2. h(x) = g(x) f(x)
3. h(x) = f(g(x))
4. h(x) = f(x) + g(x)

240. Let $S_1 = \frac{1}{3} - \frac{1}{2}\times\frac{1}{3^2} + \frac{1}{3}\times\frac{1}{3^3} - \frac{1}{4}\times\frac{1}{3^4} + \ldots$ and $S_2 = \frac{1}{4} + \frac{1}{2}\times\frac{1}{4^2} + \frac{1}{3}\times\frac{1}{4^3} + \frac{1}{4}\times\frac{1}{4^4} + \ldots$

Which of the following identities is true?

1. $3S_1 = 4S_2$
2. $4S_1 = 3S_2$
3. $S_1 + S_2 = 0$
4. $S_1 = S_2$

241. Which of the following sets are countable?

1. The set of all polynomials with rational coefficients
2. The set of all polynomials with real coefficients having rational roots
3. The set of all 2 $\times$ 2 real matrices with rational eigenvalues
4. The set of all real matrices whose row echelon form has rational entries

242. $\lim_{n\to\infty} \frac{1}{n}(1 + \sqrt{2} + \sqrt[3]{3} + \ldots + \sqrt[n]{n})$

1. is equal to 0
2. is equal to 1
3. is equal to 2
4. does not exist

243. Let (X, d) be a metric space and let f : X $\to$ X be a function such that d(f(x), f(y)) $\leq$ d(x, y) for every x, y $\in$ X. Which of the following statements is necessarily true?

1. f is continuous
2. f is injective
3. f is surjective
4. f is injective if and only if f is surjective

PART – C

244. Let (a_n) and (b_n) be two sequences of real numbers and E and F be two subsets of $\mathbb{R}$. Let E + F = {a + b: a $\in$ E, b $\in$ F}. Assume that the right hand side is well defined in each of the following statements. Which of the following statements are true?

1. $\limsup_{n\to\infty}(a_n + b_n) \le \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$
2. limsup (E + F) $\le$ limsup E + limsup F
3. $\liminf_{n\to\infty}(a_n + b_n) \le \liminf_{n\to\infty} a_n + \liminf_{n\to\infty} b_n$
4. liminf (E + F) = liminf E + limsup F

245. Let $\mathbb{R}^+$ denote the set of all positive real numbers. Suppose that $f : \mathbb{R}^+ \mapsto \mathbb{R}$ is a differentiable function. Consider the function $g(x) = e^x f(x)$. Which of the following are true?

1. If $\lim_{x\to\infty} f(x) = 0$ then $\lim_{x\to\infty} f'(x) = 0$
2. If $\lim_{x\to\infty}(f(x) + f'(x)) = 0$ then $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} \dfrac{g(x)-g(y)}{e^x - e^y} = 0$
3. If $\lim_{x\to\infty} f'(x) = 0$ then $\lim_{x\to\infty} f(x) = 0$
4. If $\lim_{x\to\infty}(f(x) + f'(x)) = 0$ then $\lim_{x\to\infty} f(x) = 0$

246. Let $A \subseteq \mathbb{R}$ and let $f : \mathbb{R}\to\mathbb{R}$ be continuous. Which of the following statements are true?

1. If A is closed then f(A) is closed
2. If A is bounded then $f^{-1}(A)$ is bounded
3. If A is closed and bounded then f(A) is closed and bounded
4. If A is bounded then f(A) is bounded

247. In which of the following cases does there exist a continuous and onto function $f : X \to Y$?

1. X = (0, 1), Y = (0, 1]
2. X = [0, 1], Y = (0, 1]
3. X = (0, 1), Y = $\mathbb{R}$
4. X = (0, 2), Y = {0, 1}

248. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that $\int_0^t f(x)\,dx = \int_t^1 f(x)\,dx,$ for every t $\in$ [0, 1]. Then which of the following are necessarily true?

1. f is differentiable on (0, 1)
2. f is monotonic on [0, 1]
3. $\int_0^1 f(x)\,dx = 1$
4. f(x) > 0 for all rationals x $\in$ [0, 1]

249. For non-negative integers k $\ge$ 1 define

$$f_k(x) = \frac{x^k}{(1+x)^2}\ \forall\, x \ge 0$$

Which of the following statements are true?

1. For each k, f_k is a function of bounded variation on compact intervals
2. For every k, $\int_0^\infty f_k(x)\,dx < \infty$
3. $\lim_{k\to\infty}\int_0^1 f_k(x)\,dx$ exists
4. The sequence of functions f_k converge uniformly on [0, 1] as $k \to\infty$

250. Let Y be a non-empty bounded, open subset of $\mathbb{R}^n$ and let $\overline{Y}$ denote its closure. Let $\{U_j\}_{j\geq 1}$ be a collection of open sets in $\mathbb{R}^n$ such that $\overline{Y} \subseteq \bigcup_{j\geq 1} U_j$. Which of the following statements are true?

1. There exist finitely many positive integers $j_1, \ldots, j_N$ such that $Y \subseteq \bigcup_{k=1}^{N} U_{j_k}$

2. There exists a positive integer N such that $Y \subseteq \bigcup_{j=1}^{N} U_j$

3. For every subsequence $j_1, j_2, \ldots$ we have $Y \subseteq \bigcup_{k=1}^{\infty} U_{j_k}$

4. There exists a subsequence $j_1, j_2, \ldots$ such that $Y = \bigcup_{k=1}^{\infty} U_{j_k}$

251. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a bounded function such that for each $t \in \mathbb{R}$, the functions g_t and h_t given by $g_t(y) = f(t, y)$ and $h_t(x) = f(x, t)$ are non decreasing functions. Which of the following statements are necessarily true?
1. $k(x) = f(x, x)$ is a non-decreasing function
2. Number of discontinuities of f is at most countably infinite
3. $\lim_{(x,y)\to(+\infty, +\infty)} f(x,y)$ exists
4. $\lim_{(x,y)\to(+\infty, -\infty)} f(x,y)$ exists

252. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be a C^1 function with $f(0, 0, 0) = (0, 0)$. Let A denote the derivative of f at (0, 0, 0). Let $g : \mathbb{R}^3 \to \mathbb{R}$ be the function given by $g(x, y, z) = xy + yz + zx + x + y + z$.
Let $h : \mathbb{R}^3 \to \mathbb{R}^3$ be the function defined by $h = (f, g)$.
In which of the following cases, will the function h admit a differentiable inverse in some open neighbourhood of (0, 0, 0)?

1. $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
2. $A = \begin{pmatrix} 2 & 2 & 2 \\ 6 & 5 & 2 \end{pmatrix}$
3. $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$
4. $A = \begin{pmatrix} 4 & 2 & 4 \\ 0 & 3 & 2 \end{pmatrix}$

JUNE-2022

PART – B

253. Let $a_n = n + n^{-1}$. Which of the following is true for the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_{n+1}}{n!}$?

1. It does not converge
2. It converges to $e^{-1} - 1$
3. It converges to e^{-1}
4. It converges to $e^{-1} + 1$

254. Consider the series $\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log_e n)^c}$. For which values of a, b, c $\in \mathbb{R}$, does the series NOT converge?

1. $|a| < 1$, b, c $\in \mathbb{R}$
2. a = 1, b > 1, c $\in \mathbb{R}$
3. a = 1, $1 \geq b$, c < 1
4. a = -1, $b \geq 0$, c > 0

255. Suppose $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ are two bold sequences of real numbers. Which of the following is true?

1. $\limsup_{n\to\infty}(a_n+(-1)^n b_n)=\limsup_{n\to\infty} a_n+\left|\limsup_{n\to\infty} b_n\right|$
2. $\limsup_{n\to\infty}(a_n+(-1)^n b_n)\leq\limsup_{n\to\infty} a_n+\limsup_{n\to\infty} b_n$
3. $\limsup_{n\to\infty}(a_n+(-1)^n b_n)\leq\limsup_{n\to\infty} a_n+\left|\limsup_{n\to\infty} b_n\right|+\left|\liminf_{n\to\infty} b_n\right|$
4. $\limsup_{n\to\infty}(a_n+(-1)^n b_n)$ may not exist

256. Let $f_n : [0, 1] \to \mathbb{R}$ be given by $f_n(t) = (n + 2)(n + 1)t^n(1 - t)$, for all t in [0, 1]. Which of the following is true?
1. The sequence (f_n) converges uniformly
2. The sequence (f_n) converges pointwise but not uniformly
3. The sequence (f_n) diverges on [0, 1)
4. $\lim_{n\to\infty}\int_0^1 f_n(t)\,dt=\int_0^1\lim_{n\to\infty} f_n(t)\,dt$

257. Let X, Y be defined by $X=\{(x_n)_{n\geq1}:\limsup_{n\to\infty} x_n=1\, where\, x_n\in\{0,1\}\}$ and $Y=\{(x_n)_{n\geq1}:\lim_{n\to\infty} x_n$ does not exists where $x_n \in \{0, 1\}\}$. Which of following is true
1. X, Y are countable
2. X is countable and Y is uncountable
3. X is uncountable and Y is countable
4. X, Y are uncountable

258. Let us define a sequence $(a_n)_{n\in\mathbb{N}}$ of real numbers to be a Fibonacci-like sequence if $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. What is the dimension of the $\mathbb{R}$ vector space of Fibonacci-like sequences?
1. 1
2. 2
3. infinite and countable
4. infinite and uncountable

259. Let D denote a proper dense subset of a metric space X. Suppose that $f : D \to \mathbb{R}$ is a uniformly continuous function. For $p \in X$, let $B_n(p)$ denote the set $\left\{x\in D: d(x,p)<\frac{1}{n}\right\}$.

Consider $W_p=\bigcap_n \overline{f(B_n(p))}$. Which of the following statements is true?
1. W_p may be empty for some p in X.
2. W_p is not empty for every p in X and is contained in f(D).
3. W_p is a singleton for every p.
4. W_p is empty for some p and singleton for some p.

260. Let X be a connected metric space with atleast two points. Which of the following is necessarily true?
1. X has finitely many points
2. X has countably many points but is not finite
3. X has uncountably many points
4. No such X exists

PART – C

261. Consider the following assertions:
S_1: $e^{\cos(t)} \neq e^{2022\sin(t)}$ for all $t \in (0, \pi)$.

S_2: For each x > 0, there exists a t ∈ (0, x) such that $x = \log_e(1 + xe^t)$.
S_3: $e^{|\sin(x)|} \le e^{|x|}$ for all x ∈ (-1, 1).
Which of the above assertions are correct?
1. Only S_1
2. Only S_3
3. Only S_1 and S_2
4. Only S_2 and S_3

262. Let $\Omega = \bigcup_{i=1}^{5} (i, i+1) \subset \mathbb{R}$ and $f : \Omega \to \mathbb{R}$ be a differentiable function such that $f'(x) = 0$ for all x ∈ Ω and let $g : \mathbb{R} \to \mathbb{R}$ be any function. Which of the following statements are true?
1. If g is continuous, then $(g \circ f)(\Omega)$ is a compact set in $\mathbb{R}$.
2. If g is differentiable and $g'(x) > 0$ for all $x \in \mathbb{R}$, then $(g \circ f)(\Omega)$ has precisely 5 elements.
3. If g is continuous and surjective, then $(g \circ f)(\Omega) \cap \mathbb{Q} \ne \emptyset$.
4. If g is differentiable, then $\{e^x : x \in (g \circ f)(\Omega)\}$ does not contain any non-empty open interval.

263. Let [x] denote the integer part of x for any real number x. Which of the following sets have non-zero Lebesgue measure?
1. $\{x \in [1, \infty) : \lim_{n\to\infty}[x]^n \text{ exists}\}$
2. $\{x \in [1, \infty) : \lim_{n\to\infty}[x^n] \text{ exists}\}$
3. $\{x \in [1, \infty) : \lim_{n\to\infty}n[x]^n \text{ exists}\}$
4. $\{x \in [1, \infty) : \lim_{n\to\infty}[1-x]^n$ exists

264. Let (X, d) be a finite non-singleton metric space. Which of the following statements are true?
1. There exists A ⊆ X such that A is not open in X.
2. X is compact.
3. X is not connected.
4. There exists a function $f : X \to \mathbb{R}$ such that f is not continuous.

265. What is the largest positive real number δ such that whenever |x – y| < δ, we have $|\cos x - \cos y| < \sqrt{2}$?
1. $\sqrt{2}$
2. $\frac{3}{2}$
3. $\frac{\pi}{2}$
4. 2

266. Let a, b ∈ ℝ such that a < b and let f : (a, b) → ℝ be a continuous function. Which of the following statements are true?
1. If f is uniformly continuous then there exist α ≥ 0 and β ≥ 0 satisfying |f(x) – f(y)| ≤ α|x – y| + β, for all x, y in (a, b).
2. For every c, d such that [c, d] ⊆ (a, b), if f restricted to [c, d] is uniformly continuous then f is uniformly continuous.
3. If f is strictly increasing and bounded than f is uniformly continuous.
4. If f is uniformly continuous then it maps Cauchy sequences into convergent sequences.

267. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} (x-y)^2 \sin\dfrac{1}{(x-y)} & if \quad x \ne y \\ 0 & if \quad x = y \end{cases}$$

Which of following statements are true

1. f is continuous at (0, 0)
2. The partial derivative f_x does not exist at (0, 0)

3. The partial derivative f_x is continuous at (0, 0)
4. f is differentiable at (0, 0)

268. Which of the given sequences (a_n) satisfy following identity?

$$\limsup_{n\to\infty} a_n = -\liminf_{n\to\infty} a_n$$

1. $a_n = \frac{1}{n}$ for all n
2. $a_n = (-1)^n \left(n + \frac{1}{n}\right)$ for all n
3. $a_n = 1 + \frac{(-1)^n}{n}$ for all n
4. (a_n) is an enumeration of all rational numbers in (-1, 1)

269. For $\alpha \geq 0$, define $a_n = \frac{1 + 2^\alpha + ... + n^\alpha}{n^{\alpha+1}}$. What is the value of $\lim_{n\to\infty} a_n$?

1. The limit does not exist
2. $\frac{1}{\alpha^2 + 1}$
3. $\frac{1}{\alpha + 1}$
4. $\frac{1}{\alpha^2 + \alpha + 1}$

270. Consider the function f : $\mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x^{\frac{1}{3}} y^{\frac{1}{3}}$ (x, y $\in \mathbb{R}$).
Which of following statements are true?
1. The directional derivative of f exists at (0, 0) in some direction
2. The partial derivative f_x does not exist at (0, 0)
3. f is continuous at (0, 0)
4. f is not differentiable at (0, 0)

JUNE-2023

PART – B

271. Consider $\mathbb{R}$ with the usual topology. Which of the following assertions is correct?
1. A finite set containing 33 elements has atleast 3 different Hausdorff topologies.
2. Let X be a non-empty finite set with a Hausdorff topology. Consider X $\times$ X with the product topology. Then, every function f : X $\times$ X $\to \mathbb{R}$ is continuous.
3. Let X be a discrete topological space having infinitely many elements. Let f : $\mathbb{R} \to$ X be a continuous function and g : X $\to \mathbb{R}$ be any non-constant function. Then the range of g $\circ$ f contains at least 2 elements.
4. If a non-empty metric space X has a finite dense subset, then there exists a discontinuous function f : X $\to \mathbb{R}$.

272. How many real roots does the polynomial $x^3 + 3x - 2023$ have?
1. 0
2. 1
3. 2
4. 3

273. Suppose S is an infinite set. Assuming that the axiom of choice holds, which of the following is true?
1. S is in bijection with the set of rational numbers.
2. S in in bijection with the set of real numbers.
3. S is in bijection with S $\times$ S.

4. S is in bijection with the power set of S.

274. Consider the series $\sum_{n=1}^{\infty} a_n$, where $a_n = (-1)^{n+1}(\sqrt{n+1}-\sqrt{n})$. Which of the following statements is true?
1. The series is divergent.
2. The series is convergent.
3. The series is conditionally convergent.
4. The series is absolutely convergent.

275. Let x, y ∈ [0, 1] be such that x ≠ y. Which of the following statements is true for every ∈ > 0?
1. There exists a positive integer N such that |x – y| < 2^n ∈ for every integer.
2. There exists a positive integer N such that 2^n ∈ < |x – y| for every integer.
3. There exists a positive integer N such that |x – y| < 2^{-n} ∈ for every integer.
4. For every positive integer N, |x – y| < 2^{-n} ∈ for some integer n ≥ N.

276. Which one of the following functions is uniformly continuous on the interval (0, 1)?
1. $f(x) = \sin\frac{1}{x}$
2. $f(x) = e^{-1/x^2}$
3. $f(x) = e^x \cos\frac{1}{x}$
4. $f(x) = \cos x \cos\frac{\pi}{x}$

277. Which of the following assertions is correct?
1. $\lim_n \sup e^{\cos\left(\frac{n\pi+(-1)^n 2e}{2n}\right)} > 1.$
2. $\lim_n e^{\log_e\left(\frac{n\pi^2+(-1)^n e^2}{7n}\right)}$ does not exist.
3. $\lim_n \inf\ e^{\sin\left(\frac{n\pi+(-1)^n 2e}{2n}\right)} < \pi.$
4. $\lim_n e^{\tan\left(\frac{n\pi^2+(-1)^n e^2}{7n}\right)}$ does not exist.

PART – C

278. Which of the following statements are correct?
1. The set of open right half-planes is a basis for the usual (Euclidean) topology on $\mathbb{R}^2$.
2. The set of lines parallel to Y-axis is a basis for the dictionary order topology on $\mathbb{R}^2$.
3. The set of open rectangles is a basis for the usual (Euclidean) topology on $\mathbb{R}^2$.
4. The set of line segments (without end points) parallel to Y-axis is a basis for the dictionary order topology on $\mathbb{R}^2$.

279. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x^2 - y^3$.
Which of the following statements are true?
1. There is no continuous real-valued function g defined on any interval of $\mathbb{R}$ containing 0 such that f(x, g(x)) = 0.
2. There is exactly one continuous real-valued function g defined on an interval of $\mathbb{R}$ containing 0 such that f(x, g(x)) = 0.
3. There is exactly one differentiable real-valued function g defined on an interval of $\mathbb{R}$ containing 0 such that f(x, g(x)) = 0.

4. There are two distinct differentiable real-valued functions g on an interval of ℝ containing 0 such that f(x, g(x)) = 0.

280. Which of the following are true?

1. For $n \geq 1$, the sequence of functions $f_n : (0, 1) \to (0, 1)$ defined by $f_n(x) = x^n$ is uniformly convergent.
2. For $n \geq 1$, the sequence of functions $f_n: (0, 1) \to (0, 1)$ defined by $f_n(x) = \dfrac{x^n}{\log(n+1)}$ is uniformly convergent.
3. For $n \geq 1$, the sequence of functions $f_n: (0, 1) \to (0, 1)$ defined by $f_n(x) = \dfrac{x^n}{1+x^n}$ is uniformly convergent.
4. For $n \geq 1$, the sequence of functions $f_n: (0, 1) \to (0, 1)$ defined by $f_n(x) = \dfrac{x^n}{1+nx^n}$ is not uniformly convergent.

281. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \dfrac{1}{4} + x - x^2$. Given $a \in \mathbb{R}$, let us define the sequence $\{x_n\}$ by $x_0 = a$ and $x_n = f(x_{n-1})$ for $n \geq 1$.
Which of the following statements are true?

1. If a = 0, then the sequence $\{x_n\}$ converges to $\dfrac{1}{2}$.
2. If a = 0, then the sequence $\{x_n\}$ converges to $-\dfrac{1}{2}$.
3. The sequence $\{x_n\}$ converges for every $a \in \left(-\dfrac{1}{2}, \dfrac{3}{2}\right)$, and it converges to $\dfrac{1}{2}$.
4. If a = 0, then the sequence $\{x_n\}$ does not converge.

282. Define $f : \mathbb{R}^4 \to \mathbb{R}$ by f(x, y, z, w) = xw – yz. Which of the following statements are true?

1. f is continuous
2. if $U = \{(x, y, z, w) \in \mathbb{R}^4: xy + zw = 0, x^2 + z^2 = 1, y^2 + w^2 = 1\}$, then f is uniformly continuous on U.
3. if $V = \{(x, y, z, w) \in \mathbb{R}^4 : x = y, z = w\}$, then f is uniformly continuous on V.
4. if $W = \{(x, y, z, w) \in \mathbb{R}^4: 0 \leq x + y + z + w \leq 1\}$, then f is unbounded on W.

283. Let μ denote the Lebesgue measure on ℝ and μ^* be the associated Lebesgue outer measure. Let A be a non-empty subset of [0, 1]. Which of the following statements are true?

1. If both interior and closure of A have the same outer measure, then A is Lebesgue measurable.
2. If A is open, then A is Lebesgue measureable and $\mu(A) > 0$.
3. If A is not Lebesgue measurable, then the set of irrationals in A must have positive outer measure.
4. If $\mu^*(A) = 0$, then A has empty interior.

284. Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} \sin(\pi/x) & when\, x \neq 0 \\ 0 & when\, x = 0. \end{cases}$$

On which of the following subsets of ℝ, the restriction of f is a continuous function?

1. [-1, 1]
2. (0, 1)
3. $\{0\} \cup \{(1/n): n \in \mathbb{N}\}$
4. $\{1/2^n : n \in \mathbb{N}\}$

285. Let $\{x_n\}$ be a sequence of positive real numbers. If $\sigma_n = \frac{1}{n}(x_1 + x_2 + \ldots x_n)$, then which of the following are true? (Here lim sup denotes the limit supremum of a sequence.)

1. If lim sup$\{x_n\} = \ell$ and $\{x_n\}$ is decreasing, then lim sup$\{\sigma_n\} = \ell$.
2. lim sup$\{x_n\} = \ell$ if and only if lim sup $\{\sigma_n\} = \ell$.
3. If $\limsup \left\{ n\left(\frac{x_n}{x_{(n+1)}} - 1 \right) \right\} < 1,$ then $\sum_n x_n$ is convergent.
4. If $\limsup \left\{ n\left(\frac{x_n}{x_{(n+1)}} - 1 \right) \right\} < 1,$ then $\sum_n x_n$ is divergent.

286. Under which of the following conditions is the sequence $\{x_n\}$ of real numbers convergent?

1. The subsequences $\{x_{(2n+1)}\}$, $\{x_{2n}\}$ and $\{x_{3n}\}$ are convergent and have the same limit.
2. The subsequences $\{x_{(2n+1)}$, $\{x_{2n}\}$ and $\{x_{3n}\}$ are convergent.
3. The subsequences $\{x_{kn}\}_n$ are convergent for every $k \geq 2$.
4. $\lim_n |x_{(n+1)} - x_n| = 0.$

287. Consider the following statements:

(a) Let f be a continuous function on $[1, \infty)$ taking non-negative values such that $\int_1^\infty f(x)\,dx$ converges. Then $\sum_{n\geq 1} f(n)$ converges.

(b) Let f be a function on $[1, \infty)$ taking non-negative values such that $\int_1^\infty f(x)\,dx$ converges. Then $\lim_{x\to\infty} f(x) = 0.$

(c) Let f be a continuous, decreasing function on $[1, \infty)$ taking non-negative values such that $\int_1^\infty f(x)\,dx$ does not converge. Then $\sum_{n\geq 1} f(n)$ does not converge.

Which of the following options are true?

1. (a), (b) and (c) are all true.
2. Both (a) and (b) are false.
3. (c) is true.
4. (b) is true.

<u>DECEMBER-2023</u>

<u>PART – B</u>

288. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that f and its derivative f' have no common zeros in [0, 1]. Which one of the following statements is true?

1. f never vanishes in [0, 1].
2. f has atmost finitely many zeros in [0, 1].
3. f has infinitely many zeros in [0, 1].
4. $f(1/2) = 0$.

289. Let f(x) be a cubic polynomial with real coefficients. Suppose that f(x) has exactly one real root and that this root is simple. Which one of the following statements holds for ALL anti-derivatives F(x) of f(x)?

1. F(x) has exactly one real root.
2. F(x) has exactly four real roots.
3. F(x) has atmost two real roots.
4. F(x) has atmost one real root.

290. Consider the following infinite series:

(a) $\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{\sqrt{n}}$, (b) $\sum_{n=1}^{\infty} \log\left(1+\frac{1}{n^2}\right)$.

Which one of the following statements is true?
1. (a) is convergent, but (b) is not convergent.
2. (a) is not convergent, but (b) is convergent.
3. Both (a) and (b) are convergent.
4. Neither (a) nor (b) is convergent.

291. Let f : $\mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} (1-x)^2 \sin(x^2), & x \in (0,1). \\ 0, & otherwise \end{cases}$$

and f′ be its derivative. Let

S = {c ∈ $\mathbb{R}$: f′(x) ≤ cf(x) for all x ∈ $\mathbb{R}$}.

Which one of the following is true?

1. S = ∅
2. S ≠ ∅ and S is a proper subset of (1, ∞)
3. (2, ∞) is a proper subset of S
4. S ∩ (0, 1) ≠ ∅

292. Consider the following subset of $\mathbb{R}$:

U = {x ∈ $\mathbb{R}$: $x^2 - 9x + 18 \le 0$, $x^2 - 7x + 12 \le 0$}.

Which one of the following statements is true?
1. inf U = 5.
2. inf U = 4.
3. inf U = 3.
4. inf U = 2.

293. Let X be a non-empty finite set and Y = {$f^{-1}(0)$: f is a real-valued function on X}.

Which one of the following statements is true?
1. Y is an infinite set
2. Y has $2^{|X|}$ elements
3. There is a bijective function from X to Y
4. There is a surjective function from X to Y

294. Consider the sequence $(a_n)_{n\ge1}$, where $a_n = \cos\left((-1)^n \frac{n\pi}{2} + \frac{n\pi}{3}\right)$. Which one of the following statements is true?

1. $\limsup_{n\to\infty} a_n = \frac{\sqrt{3}}{2}$.
2. $\limsup_{n\to\infty} a_{2n} = 1$.
3. $\limsup_{n\to\infty} a_{2n} = \frac{1}{2}$.
4. $\limsup_{n\to\infty} a_{3n} = 0$.

PART – C

295. Suppose that f : [-1, 1] → $\mathbb{R}$ is continuous. Which of the following imply that f is identically zero on [-1, 1]?

1. $\int_{-1}^{1} f(x)x^n dx = 0$ for all n ≥ 0.
2. $\int_{-1}^{1} f(x)p(x)dx = 0$ for all real polynomials p(x).

3. $\int_{-1}^{1} f(x)x^n dx = 0$ for all n ≥ 0 odd.

4. $\int_{-1}^{1} f(x)x^n dx = 0$ for all n ≥ 0 even.

296. For a real number λ, consider the improper integrals

$$I_\lambda = \int_0^1 \frac{dx}{(1-x)^\lambda}, K_\lambda = \int_1^\infty \frac{dx}{x^\lambda}.$$

Which of the following statements are true?

1. There exists λ such that I_λ converges, but K_λ does not converge.
2. There exists λ such that K_λ converges, but I_λ does not converge.
3. There exists λ such that I_λ, K_λ both converge.
4. There exists λ such that neither I_λ nor K_λ converges.

297. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $|f(x) - f(y)| \geq \log(1 + |x - y|)$ for all $x, y \in \mathbb{R}$. Which of the following statements are true?

1. f is necessarily one-one.
2. f need not be one-one.
3. f is necessarily onto.
4. f need not be onto.

298. Let $p : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $p(x, y) = x$. Which of the following statements are true?

1. Let $A_1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$. Then for each $\gamma \in p(A_1)$, there exists a positive real number ε such that $(\gamma - \varepsilon, \gamma + \varepsilon) \subseteq p(A_1)$.
2. Let $A_2 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$. Then for each $\gamma \in p(A_2)$, there exists a positive real number ε such that $(\gamma - \varepsilon, \gamma + \varepsilon) \subseteq p(A_2)$.
3. Let $A_3 = \{(x, y) \in \mathbb{R}^2 \mid xy = 0\}$. Then for each $\gamma \in p(A_3)$, there exists a positive real number ε such that $(\gamma - \varepsilon, \gamma + \varepsilon) \subseteq p(A_3)$.
4. Let $A_4 = \{(x, y) \in \mathbb{R}^2 \mid xy = 1\}$. Then for each $\gamma \in p(A_4)$, there exists a positive real number ε such that $(\gamma - \varepsilon, \gamma + \varepsilon) \subseteq p(A_4)$

299. Let x be a real number. Which of the following statements are true?

1. There exists an integer n ≥ 1 such that $n^2 \sin\frac{1}{n} \geq x.$
2. There exists an integer n ≥ 1 such that $n\cos\frac{1}{n} \geq x.$
3. There exists an integer n ≥ 1 such that $ne^{-n} \geq x$.
4. There exists an integer n ≥ 2 such that $n(\log n)^{-1} \geq x$.

300. For real numbers a, b, c, d, e, f, consider the function $F : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$F(x, y) = (ax + by + c, dx + ey + f)$, for $x, y \in \mathbb{R}$.

Which of the following statements are true?

1. F is continuous
2. F is uniformly continuous
3. F is differentiable
4. F has partial derivatives of all orders

301. For a differentiable surjective function $f : (0, 1) \to (0, 1)$, consider the function

$F : (0, 1) \times (0, 1) \to (0, 1) \times (0, 1)$ given by $F(x, y) = (f(x), f(y))$, $x, y \in (0, 1)$. If $f'(x) \neq 0$ for every $x \in (0, 1)$, then which of the following statements are true?

1. F is injective.
2. f is increasing.

3. For every $(x', y') \in (0, 1) \times (0, 1)$, there exists a unique $(x, y) \in (0, 1) \times (0, 1)$ such that $F(x, y) = (x', y')$.
4. The total derivative $DF(x, y)$ is invertible for all $(x, y) \in (0, 1) \times (0, 1)$.

302. Which of the following statements are true?

1. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} [x]\sin\frac{1}{x} & for\ x \neq 0, \\ 0 & for\ x = 0 \end{cases}$ has a discontinuity at 0 which is removable.
2. The function $f : [0, \infty) \to \mathbb{R}$ defined by $f(x) = \begin{cases} \sin(\log\ x) & for\ x \neq 0, \\ 0 & for\ x = 0 \end{cases}$ has a discontinuity at 0 which is NOT removable.
3. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} e^{1/x} & for\ x < 0, \\ e^{1/(x+1)} & for\ x \geq 0 \end{cases}$ has a jump discontinuity at 0.
4. Let $f, g : [0, 1] \to \mathbb{R}$ be two functions of bounded variation. Then the product fg has atmost countably many discontinuities.

303. Let $(f_n)_{n\geq 1}$ be the sequence of functions defined on [0, 1] by

$$f_n(x) = x^n \log\left(\frac{1+\sqrt{x}}{2}\right).$$

Which of the following statements are true?
1. (f_n) converges pointwise on [0, 1].
2. (f_n) converges uniformly on compact subsets of [0, 1) but not on [0, 1).
3. (f_n) converges uniformly on [0, 1) but not on [0, 1].
4. (f_n) converges uniformly on [0, 1].

304. Let $\{A_n\}_{n\geq 1}$ be a collection of non-empty subsets of $\mathbb{Z}$ such that $A_n \cap A_m = \emptyset$ for $m \neq n$. If $\mathbb{Z} = \cup_{n\geq 1} A_n$, then which of the following statements are necessarily true?
1. A_n is finite for every integer $n \geq 1$.
2. A_n is finite for some integer $n \geq 1$.
3. A_n is infinite for some integer $n \geq 1$.
4. A_n is countable (finite or infinite) for every integer $n \geq 1$.

305. Let $f : [0, \infty) \to \mathbb{R}$ be the periodic function of period 1 given by
$f(x) = 1 - |2x - 1|$ for $x \in [0, 1]$.
Further, define $g : [0, \infty) \to \mathbb{R}$ by $g(x) = f(x^2)$. Which of the following statements are true?
1. f is continuous on $[0, \infty)$.
2. f is uniformly continuous on $[0, \infty)$.
3. g is continuous on $[0, \infty)$.
4. g is uniformly continuous on $[0, \infty)$.

JUNE-2024

PART – B

306. Let $(a_n)_{n\geq 1}$ be a bounded sequence in $\mathbb{R}$. Which of the following statements is FALSE?
1. If $\liminf_{n\to\infty} a_n = \limsup_{n\to\infty} a_n$, then (a_n) is convergent
2. If $\inf\{a_n \mid n \geq 1\} = \limsup_{n\to\infty} a_n$, then (a_n) is convergent

3. If sup $\{a_n \mid n \geq 1\} = \liminf\limits_{n\to\infty} a_n$, then (a_n) is constant

4. If sup $\{a_n \mid n \geq 1)$ = inf $\{a_n \mid n \geq 1\}$, then (a_n) is constant

307. What is the cardinality of the set of real solutions of $e^x + x = 1$?

1. 0
2. 1
3. Countably infinite
4. Uncountable

308. Consider the set A = $\{x \in \mathbb{Q} : 0 < (\sqrt{2} - 1)\, x < \sqrt{2} + 1\}$ as a subset of $\mathbb{R}$. Which of the following statements is true?

1. sup A = $2 + 2\sqrt{3}$
2. sup A = $3 + 2\sqrt{2}$
3. inf A = $2 + 2\sqrt{3}$
4. inf A = $3 + 2\sqrt{2}$

309. Let S = $\left\{x \in \mathbb{R} : x > 1 \text{ and } \dfrac{1-x^4}{1-x^3} > 22\right\}$. Which of the following is true about S?

1. S is empty
2. There is a bijection between S and $\mathbb{N}$
3. There is a bijection between S and $\mathbb{R}$
4. There is a bijection between S and a non-empty finite set

310. Let C be the collection of all sets S such that the power set of S is countably infinite. Which of the following statements is true?

1. There exists a non-empty finite set in C
2. There exists a countably infinite set in C
3. There exists an uncountable set in C
4. C is empty

311. Let S be a dense subset of $\mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ a given function. Define $g : S \to \mathbb{R}$ by g(x) = f(x). Which of the following statements is necessarily true?

1. If f is continuous on the set S, then f is continuous on the set $\mathbb{R}\setminus S$
2. If g is continuous, then f is continouos on the set S
3. If g is identically 0 and f is continuous on the set $\mathbb{R}\setminus S$, then f is identically 0
4. If g is identically 0 and f is continuous on the set S, then f is identically 0

312. For each $n \geq 1$ define $f_n : \mathbb{R} \to \mathbb{R}$ by

$$f_n(x) = \frac{x^2}{\sqrt{x^2 + \frac{1}{n}}}, x \in \mathbb{R}$$

where $\sqrt{\ }$ denotes the non-negative square root. Whereever $\lim\limits_{n\to\infty} f_n(x)$ exists, denote it by f(x). Which of the following statements is true?

1. There exists $x \in \mathbb{R}$ such that f(x) is not defined
2. f(x) = 0 for all $x \in \mathbb{R}$
3. f(x) = x for all $x \in \mathbb{R}$
4. f(x) = |x| for all $x \in \mathbb{R}$

PART – C

313. Define $f : \mathbb{R} \to \mathbb{R}$ by f(x) = x|x|. Which of the following statements are true?

1. f is continuous on $\mathbb{R}$
2. f is differentiable on $\mathbb{R}$
3. f is differentiable only at 0
4. f is not differentiable at 0

314. Let $(a_n)_{n\geq 1}$ be a bounded sequence of real numbers suc that $\lim_{n\to\infty} a_n$ does not exist. Let $S = \{l \in \mathbb{R}$: there exists a subsequence of (a_n) converges to $l\}$.
Which of the following statements are necessarily true?
1. S is the empty set
2. S has exactly one element
3. S has atleast two elements
4. S has to be a finite set

315. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of real numbers. For $n \geq 1$ define

$$A_n = \begin{cases} a_n, & if\ a_n > 0 \\ 0, & otherwise \end{cases} \qquad B_n = \begin{cases} a_n, & if\ a_n > 0 \\ 0, & otherwise. \end{cases}$$

Which of the followiing statements are necessarily true?
1. $A_n \to 0$ and $B_n \to 0$ as $n \to \infty$
2. If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then both $\sum_{n=1}^{\infty} A_n$ and $\sum_{n=1}^{\infty} B_n$ are absolutely convergent
3. Both $\sum_{n=1}^{\infty} A_n$ and $\sum_{n=1}^{\infty} B_n$ are convergent
4. If $\sum_{n=1}^{\infty} a_n$ is not absolutely convergent, then both $\sum_{n=1}^{\infty} A_n$ and $\sum_{n=1}^{\infty} B_n$ are divergent.

316. Consider the improper integrals

$$I = \int_{\pi/2}^{\pi} \frac{1}{\sqrt{\sin x}}\,dx$$

and, for $a \geq 0$

$$I_a = \int_{a}^{\infty} \frac{1}{x\sqrt{1+x^2}}\,dx$$

1. The integral I is convergent
2. The integral I is not convergent
3. The integral I_a converges for $a = \frac{1}{2}$ but not for a = 0
4. The integral I_a converges for all $a \geq 0$

317. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} \dfrac{y\sqrt{x^2+y^2}}{x}, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$$

Which of the following statements are true?
1. $\frac{\partial f}{\partial x}(0,0)$ exists
2. $\frac{\partial f}{\partial y}(0,0)$ exists
3. f is not continuous at (0, 0)
4. f is not differentiable at (0, 0)

318. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be a differentiable function such that (Df) (0, 0) has rank 2. Write $f = (f_1, f_2, f_3)$. Which of the following statements are necessarily true?
1. f is injective in a neighbourhood of (0, 0)
2. There exists an open neighbourhood U of (0, 0) in $\mathbb{R}^2$ such that f_3 is a function of f_1 and f_2
3. f maps an open neighbourhood of (0, 0) in $\mathbb{R}^2$ onto an open subset of $\mathbb{R}^3$
4. (0, 0) is an isolated point of $f^{-1}(\{f(0, 0)\})$

319. Let $K \subseteq \mathbb{R}$ be non-empty and $f : K \to K$ be continuous such that
$|x - y| \leq |f(x) - f(y)|\ \forall\ x, y \in K$
Which of the following statements are true?
1. f need not be surjective
2. f must be surjective if K = [0, 1]
3. f is injective and $f^{-1} : f(K) \to K$ is continuous
4. f is injective, but $f^{-1} : f(K) \to K$ need not be continuous

320. Let $(a_n)_{n\geq 1}$ be a sequence of positive real numbers. Let $b_n = \dfrac{a_n}{\max\{a_1, ..., a_n\}}, n \geq 1$
Which of the following statements are necessarily true?
1. If $\lim\limits_{n\to\infty} b_n$ exists in $\mathbb{R}$, then $\{a_n : n \geq 1\}$ is bounded
2. If $\lim\limits_{n\to\infty} b_n = 1$, then $\lim\limits_{n\to\infty} a_n$ exists in $\mathbb{R}$
3. If $\lim\limits_{n\to\infty} b_n = \dfrac{1}{2}$, then $\lim\limits_{n\to\infty} a_n$ exists in $\mathbb{R}$
4. If $\lim\limits_{n\to\infty} b_n = 0$, then $\lim\limits_{n\to\infty} a_n = 0$

321. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous and one-to-one function. Which of the following statements are necessarily true?
1. f is strictly increasing
2. f is strictly decreasing
3. f is either strictly increasing or strictly decreasing
4. f is onto

322. Let $f : [0, 1) \to [1, \infty)$ be defined by $f(x) = \dfrac{1}{1-x}$. For $n \geq 1$, let $p_n(x) = 1 + x + \ldots + x^n$. Then which of the following statements are true>
1. f(x) is not uniformly continuous on [0, 1)
2. The sequence $(p_n(x))$ converges to f(x) pointwise on [0, 1)
3. The sequence $(p_n(x))$ converges to f(x) uniformly on [0, 1)
4. The sequence $(p_n(x))$ converges to f(x) uniformly on [0, c] for every 0 < c < 1

DECEMBER – 2024

PART – B

323. For integer $n \geq 0$, *let* $f_n : [-1, 0] \to \mathbb{R}$ be defined by $f_n(x) = \dfrac{x}{(1-x)^n}$. Which of the following statements is true about the series $\sum\limits_{n=0}^{\infty} f_n$?
1. The series is neither absolutely convergent nor uniformly convergent.
2. The series is both absolutely convergent and uniformly convergent.
3. The series is absolutely convergent but not uniformly convergent.
4. The series is uniformly convergent but not absolutely convergent.

324. Consider the sequences $(a_n)_{n\geq1}$ *and* $(b_n)_{n\geq1}$ defined by $a_n = \frac{e^n + e^{-n}}{2}$ *and* $b_n = \frac{a_{n+1}}{a_n}$. Which of the following statements is true?

1. For each $x \in \mathbb{R}$, there exists an n such that $a_n > x$
2. For each $x \in \mathbb{R}$, there exists an n such that $a_n < x$
3. For each $x \in \mathbb{R}$, there exists an n such that $b_n > x$
4. For each $x \in \mathbb{R}$, there exists an n such that $b_n < x$

325. Let A, B, and C be sets. Which of the following sets is equal to A\(B\C)?

1. A \ B
2. $(A \setminus B) \cup C$
3. $A \setminus (B \cup C)$
4. $(A \setminus B) \cup (A \cap C)$

326. Let $f : [0,1] \to \mathbb{R}$ be defined by f(x) = sin(x²). Let $A = \lim_{n\to\infty}\left(\sum_{k=1}^{n} f\left(\frac{k}{n}\right) - n\int_0^1 f(x)dx\right)$.Which of the following statements is true?

1. A=0
2. A=1
3. $A = \frac{\sin(1)}{2}$
4. $A = \sin\left(\frac{1}{4}\right)$

327. Consider the power series $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}} x^n$ with coefficients in real numbers ℝ. Which of the following statements is true?

1. The radius of convergence of the series is $\frac{1}{e}$
2. The series converges at x=5
3. The series converges at x=3
4. The series converges for all x with $|x| < \frac{1}{2}$

PART-C

328. For a positive real number a, $\sqrt{a}$ denotes the positive square root of a. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = \begin{cases} \frac{x}{|x|}\sqrt{x^2+y^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$

Which of the following statements are true?

1. f is continuous at (0,0).
2. The partial derivates $\frac{\partial f}{\partial x}$ *and* $\frac{\partial f}{\partial y}$ exist at (0,0).
3. f is differentiable at (0,0).
4. f is not differentiable at (0,0).

329. Consider the function f:$\mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} + e^{xy}, & (x,y) \neq (0,0), \\ 1, & (x,y) = (0,0). \end{cases}$

Which of the following statements are true?
1. f is differentiable on $\mathbb{R}^2\backslash\{(0,0)\}$.
2. All the directional derivatives of f exist at (0,0).
3. f is differentiable on $\mathbb{R}^2$.
4. f is not continuous at (0,0).

330. Consider the function f:$\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & if \quad x\neq 0, \\ 0, & if \ \ x=0. \end{cases}$

Which of the following statements are true?

1. $\lim\limits_{x\to 0} f(x)$ exists.
2. f is continuous at 0.
3. f is differentiable at 0.
4. $\lim\limits_{x\to 0} f'(x)$ does not exist.

331. Let $(a_n)_{n\geq 1,}(b_n)_{n\geq 1}$ *and* $(c_n)_{n\geq 1}$ be sequences given by

$a_n=(-1)^n(1+e^{-n}),\ b_n=\max\{a_1,.....a_n\}$, and $C_n=\min\{a_1,........,a_n\}$.

Which of the following statements are true?

1. $(a_n)_{n\geq 1}$ does not converge.
2. $\lim\limits_{n\to\infty}\sup\ a_n=\lim\limits_{n\to\infty} b_n$
3. $\lim\limits_{n\to\infty}\inf\ a_n=\lim\limits_{n\to\infty} c_n$
4. $\lim\limits_{n\to\infty} b_n=\lim\limits_{n\to\infty} c_n$

332. For a positive integer n and a subset S of the set of positive integers, let S(n) denote the set $\{s\in S \mid s\leq n\}$. Let X be a subset of the set of positive integers such that $\lim\limits_{n\to\infty}\frac{|X(n)|}{n}=1.$ Assume that there exist pairwise disjoint subsets $X_1, X_2, \ldots\ldots\ldots\ldots X_8$ of X such that $\bigcup\limits_{i=1}^{8} X_i=X.$ Which of the following statements are true?

1. $\lim\limits_{n\to\infty}\frac{|X_i(n)|}{n}$ *exists for all* $1\leq i\leq 8.$
2. $\lim\limits_{n\to\infty}\inf\frac{|X_i(n)|}{n}\geq 0$ *for all* $1\leq i\leq 8.$
3. $\lim\limits_{n\to\infty}\sup\frac{|X_i(n)|}{n}\geq 1/8$ *for some* $1\leq i\leq 8.$
4. $\lim\limits_{n\to\infty}\sup\frac{|X_i(n)|}{n}<1/8$ *for all* $1\leq i\leq 8.$

333. Which of the following statements are true?

1. Let x,y$\in\mathbb{R}$ with x<y. Then there exists $r\in\mathbb{Q}$ such that $x<\frac{2^{2024}r}{e}<y.$
2. Let $(a_n)_{n\geq 2}$ be a sequence of positive real numbers. If there exists a positive real number L such that $\lim\limits_{n\to\infty}\sup\frac{a_n}{\log\ n}=L,$ then $\lim\limits_{n\to\infty}\sup a_n>\infty.$
3. The set of all finite subsets of $\mathbb{Q}$ is countably infinite.
4. The set of continuous functions from $\mathbb{R}$ to the set {0,1} is infinite.

334. Let g:$\mathbb{R} \to \mathbb{R}$ be a continuous function. Define $f(x) = \int_0^x (x-t)g(t)dt, x \in \mathbb{R}$. Which of the following statements are true?

1. f(0)=0
2. f'(0) exists and f'(0)=0.
3. f''(0) exists and f''(0)=g(0)
4. f''(0) exists but f''(0) $\neq$ g(0).

335. Let g:$\mathbb{R} \to \mathbb{R}$ be a continous function such that f(x) = 0 for all $x \le 0$ and for all $x \ge 1$.

Define $F(x) = \sum_{n=-\infty}^{\infty} f(x+n), \ x \in \mathbb{R}$. Which of the following statements are true?

(1) F is bounded.
(2) F is continuous on $\mathbb{R}$
(3) F is uniformly continuous on $\mathbb{R}$
(4) F is not uniformly continuous on $\mathbb{R}$.

336. Let f:[0,1] $\to \mathbb{R}$ be a monotonic function. Which of the following statements are true?
(1) f is Riemann integrable on [0,1].
(2) The set of discontinuities of f cannot contain a non-empty open set.
(3) f is Lebesgue measurable function.
(4) f is a Borel measurable function.

337. For each positive integer n, define $f_n : [0,1] \to \mathbb{R}$ by $f_n(x) = nx(1-x)^n$. Which of the following statements are true?

1. $(f_n)_{n\ge1}$ does not converge pointwise on [0,1].
2. $(f_n)_{n\ge1}$ converges pointwise to a continuous function on [0,1].
3. $(f_n)_{n\ge1}$ converges pointwise to a discontinuous function on[0,1].
4. $(f_n)_{n\ge1}$ does not converge uniformly on[0,1].

JUNE – 2025

PART – B

338. Let f :$\mathbb{R} \setminus \mathbb{Q} \to \mathbb{R} \setminus \mathbb{Q}$ be the function defined as $f(x) = \frac{3x+2}{4x+3}$.

Let $x_1 \in \mathbb{R} \setminus \mathbb{Q}$. For n ≥ 1, define $x_{n+1} = f(x_n)$. Suppose that the sequence $(x_n)_{n\ge1}$ converges to a real number ℓ. Which of the following statements is true?

(1) If ℓ is positive, the $\ell = \frac{\sqrt{3}}{2}$.
(2) If ℓ is positive, then $\ell = \frac{1}{\sqrt{2}}$.
(3) If ℓ is negative, then $\ell = -\frac{\sqrt{3}}{2}$.
(4) If ℓ is negative, then $\ell = -\frac{1}{2}$.

339. Let p, q be non-negative integers. Consider the following statements:
(A) There is an integer k ≥ 1 such that p + k = q.
(B) There is an integer k ≥ 1 such that q + k = p

Which of the following statements is true?
(1) There exist non-negative integers p, q such that both (A) and (B) are true.
(2) Both (A) and (B) are false if and only if p = q
(3) For all non-negative integers p and q, (A) or (B) is true
(4) There exists p ≠ q such that both (A) and (B) are false

340. Let $f(x) = x\log_e\left(1+\frac{1}{x}\right)$ for x ∈ (0, ∞). Which of the following statements is true?

(1) f is unbounded (2) f is increasing

(3) $\lim_{x\to\infty} f(x) = 2$ (4) f is decreasing

341. For each n ≥ 1, let f_n $[0,1] \to \mathbb{R}$ be defined as

$$f_n(x) = \begin{cases} nx & if\ x \in \left[0, \frac{1}{n}\right], \\ 2-nx & if\ x \in \left(\frac{1}{n}, \frac{2}{n}\right], \\ 0 & if\ x \in \left(\frac{2}{n}, 1\right]. \end{cases}$$

Which of the following statements is true?

(1) $(f_n)_{n\geq 1}$ converges uniformly on [0, 1] to a continuous function f.

(2) $(f_n)_{n\geq 1}$ converges pointwise on [0, 1] to a discontinuous function f.

(3) $(f_n)_{n\geq 1}$ converges pointwise on [0, 1] to a continuous function f.

(4) $(f_n)_{n\geq 1}$ does not converge pointwise on [0, 1].

342. Let A, B be non-empty subsets of ℕ with cardinality |A| ≥ 2. Let S_1 = {f : A → B | f is one-to-one} and S_2 = {g : B → A | g is onto}.
Which of the following statements is true?

(1) If $A \subsetneq B$ and B is finite, then there is a one-to-one map from S_2 to S_1.

(2) If B = ℕ, then there exists a one-to-one map from S_2 to B.

(3) If B = ℕ and A is finite, then there exists a one-to-one map from B to S_1.

(4) If A is finite, then S_2 is finite for any B.

343. Let

A = $\left\{\frac{p}{q} \in (0,1) : p \in \mathbb{N},\ q = 2^n \text{ for some } n \in \mathbb{N} \cup \{0\},\ \gcd(p, q) = 1\right\}$

B = $\left\{\frac{p}{q} \in (0,1) : p \in \mathbb{N},\ q = 2^n 5^m \text{ for some } n, m \in \mathbb{N} \cup \{0\},\ \gcd(p, q) = 1\right\}$

C = $\left\{\frac{p}{q} \in (0,1) : \frac{p}{q} \text{ has terminating decimal expansion}\right\}$

be subsets of (0, 1). Which of the following statements is true?

(1) $A \subsetneq C$ and $B \subsetneq C$ (2) $A \subsetneq C \subsetneq B$

(3) $A \subsetneq B \subsetneq C$ (4) $A \subsetneq B = C$

PART-C

344. Let f and g be real-valued Riemann integrable functions on [a, b] such that g([a, b]) ⊆ [a, b]. Which of the following statements are necessarily true?

1. The composition $f \circ g$ is Riemann integrable.

2. If $g(x) \neq 0$ for each $x \in [a,b]$, then $\frac{f}{g}$ is Riemann integrable.

3. The positive square root $\sqrt{f^2 + g^2}$ is Riemann integrable.

4. The composition $f \circ g$ is Riemann integrable, if both f and g are continuous

345. Let C[0, 1] be the ℝ–vector space of real valued continuous functions equipped with the norm $\|f\| = \sup_{x \in [0,1]} |f(x)|$.

Let $T : C[0,1] \to C[0,1]$ be defined as

T(f) (x)= $\int_0^x f(t)dt, \; for\, x \in [0,1]$

Let $T^n = T \circ T \circ \ldots \circ T$ (n times). Which of the following statements are true?

1. There exists $\alpha \in (0,1)$ such that for all f, g $\in C[0,1], \|T(f) - T(g)\| \leq \alpha \|f - g\|$.
2. There exists $\alpha \in (0,1)$ such that for all f, g $\in C[0,1], \|T^2(f) - T^2(g)\| \leq \alpha \|f - g\|$.
3. The set $\{f \in C[0,1] : T(f) = f\}$ is a singleton set.
4. $\|T^n\| \to \infty \; as \; n \to \infty$.

346. Let f be a bounded, twice continuously differentiable real-valued function on $(0, \infty)$ such that $f''(x) \geq 0$ for all $x \in (0, \infty)$. Which of the following statements are true?

1. $f'(x) \leq 0$ for all x>0
2. $\lim_{x \to \infty} f'(x) = 0$
3. $\lim_{x \to \infty} x f'(x)$ need not exist.
4. $\lim_{x \to \infty} x f'(x) = 0$

347. Consider the sequence $(s_n)_{n \geq 1}$ and $(t_n)_{n \geq 1}$ defined by

$$s_n = \sum_{k=0}^{n} \frac{1}{(k!)^2} \; and \; t_n = \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{(n)^k}.$$

Which of the following statements are true?

1. $\limsup_{n \to \infty} t_n \leq \limsup_{n \to \infty} s_n$
2. $\limsup_{n \to \infty} t_n \leq e$
3. $\liminf_{n \to \infty} s_n \geq e^2$
4. $\liminf_{n \to \infty} t_n \geq e$

348. Let μ denote the Lebesgue measure on ℝ. Suppose that f is a non-negative Lebesgue measurable function on ℝ. Let $0 = a_0 < a_1 < a_2 < \cdots$ be an unbounded sequence such that $a_{n+1} \leq c a_n$ for some real number c and for all $n \geq 1$. Let $A_k = \{x \in \mathbb{R} \mid a_k \leq f(x) < a_{k+1}\}$ for each $k \geq 0$. Which of the following statements are true?

1. If f is Lebsgue integrable on ℝ , then $\sum_{k \geq 0} a_k \mu(A_k)$ is finite.
2. If $\sum_{k \geq 0} a_k \mu(A_k)$ is finite, then f is Lebesgue integrable on ℝ.
3. If $\sum_{k \geq 0} a_k \mu(A_k)$ is finite and $f(x) \geq \alpha$ for all $x \in \mathbb{R}$, then f is Lebesgue integrable on ℝ.

4. If $\sum_{k\geq 0} a_k \mu(A_k)$ is finite and f is bounded, then f is Lebesgue integrable on ℝ.

349. Let p: ℝ → ℝ be a non-constant polynomial. Which of the following statements are true?
1. The pre-image of a compact set under p is a compact set.
2. The pre-image of a connected set under p is a connected set.
3. Every point x ∈ ℝ has an open neighbourhood U_x such that the restriction $p|U_x$ is a homeomorphism onto an open set in ℝ.
4. The image of a bounded set under p is a bounded set.

350. For each $n \geq 1$, let f_n : ℝ → ℝ be a function defined by $f_n(x) = \frac{e^{-n^2x^2}}{n}$.

Which of the following statements are true?
1. $(f_n)_{n\geq 1}$ converges uniformly to 0 on ℝ, and $(f_n')_{n\geq 1}$ converges uniformly to 0 on the interval (-M, M) for some positive real number M.
2. $(f_n)_{n\geq 1}$ converges uniformly to 0 on ℝ, and $(f_n')_{n\geq 1}$ converges pointwise to 0 on ℝ.
3. $(f_n)_{n\geq 1}$ converges uniformly to 0 on ℝ, and $(f_n')_{n\geq 1}$ does not converges pointwise to 0 on ℝ.
4. $(f_n)_{n\geq 1}$ converges pointwise to 0 on ℝ but not uniformly on ℝ.

351. What is the value of the limit

$\lim_{n\to\infty} \frac{1}{n}[(n+1)(n+2)\cdots(n+n)]^{1/n}$?

1. $\frac{2}{e}$
2. $\frac{4}{e}$
3. $\log_e 2 - 1$
4. $2\log_e 2 - 1$

ANSWERS

1. (1)	2. (4)	3. (3)	4. (2)	5.	6. (3)
7. (4)	8. (1,3)	9. (1,3,4)	10. (3,4)	11.	12. (4)
13. (3,4)	14. (3,4)	15. (1,2)	16. (3,4)	17. (1,2,3)	18. (2,4)
19. (3)	20. (3)	21. (4)	22. (2)	23. (2)	24. (2)
25. (1)	26. (3)	27. (1,2)	28. (1,2,4)	29. (1,2,3)	30. (1,2)
31. (1,2)	32. (2,3,4)	33. (1)	34. (1,2,4)	35. (3)	36. (3)
37. (1)	38. (1)	39. (4)	40. (3)	41. (2,4)	42. (3,4)
43. (2,3)	44. (1,4)	45. (1,3,4)	46. (1,4)	47. (2,4)	48. (3)
49. (3)	50. (2)	51. (1)	52. (4)	53. (2)	54. (1,2,3,4)
55. (1,3)	56. (3,4)	57. (1,2)	58. (1,3)	59. (3,4)	60. (1,2)
61. (1,2,3)	62. (2,4)	63. (4)	64. (3,4)	65. (3,4)	66. (1)
67. (1)	68. (1)	69. (2)	70. (3)	71. (1)	72. (1,4)
73. (3,4)	74. (3,4)	75. (2,3)	76. (3,4)	77. (2,4)	78. (1,4)
79. (2,4)	80. (2)	81. (1)	82. (2)	83. (1)	84. (2)
85. (3)	86. (1,2)	87. (1)	88. (1,4)	89. (1,2,3)	90. (1,2,4)
91. (1,2,4)	92. (1,2)	93. (1,3,4)	94. (1,3)	95. (1)	96. (1,4)
97. (1)	98. (4)	99. (3)	100. (3)	101. (3)	102. (1)
103. (3)	104. (2)	105. (2,4)	106. (2,3)	107. (3)	108. (1,2,3)
109. (3,4)	110. (3,4)	111. (2)	112. (3)	113. (3)	114. (2)
115. (3)	116. (2)	117. (2)	118. (3)	119. (2)	120. (2)
121. (3)	122. (2)	123. (1)	124. (1,3,4)	125. (3,4)	126. (3,4)
127. (4)	128. (2,3,4)	129. (2)	130. (1,2,3)	131. (1,4)	132. (1,2,3,4)
133. (1,3)	134. (2)	135. (4)	136. (2)	137. (1)	138. (2)
139. (3)	140. (3)	141. (3)	142. (2)	143. (1,4)	144. (1,2, 4)
145. (1,2)	146. (1,2)	147. (1,4)	148. (1,2)	149. (4)	150. (1,3,4)
151. (2)	152. (1,3)	153. (3)	154. (2)	155. (4)	156. (3)
157. (4)	158. (2)	159. (2,3)	160. (1,4)	161. (3,4)	162. (1,2)
163. (1,2,3,4)	164. (2,3)	165. (1,2,4)	166. (1,2,3)	167. (2,4)	168. (2)
169. (2)	170. (4)	171. (2)	172. (2)	173. (1)	174. (2,3)
175. (2,4)	176. (1,2,3,4)	177. (1,2)	178. (2,3)	179. (2,3)	180. (1,3)
181. (1,2,3)	182. (3,4)	183. (1,2,3,4)	184. (2)	185. (3)	186. (3)
187. (2)	188. (3)	189. (2)	190. (1,2,4)	191. (2,3,4)	192. (3)
193. (1,4)	194. (1,3,4)	195. (4)	196. (1)	197. (4)	198. (3,4)
199. (1,2,4)	200. (3,4)	201. (1,2,3,4)	202. (3)	203. (3)	204. (4)
205. (4)	206. (4)	207. (3)	208. (2,3)	209. (1,4)	210. (1,4)
211. (1,4)	212. (1,3)	213. (2)	214. (2,3,4)	215. (2,3,4)	216. (2)
217. (1,2,3)	218. (1,3)	219. (1)	220. (2)	221. (3)	222. (3)
223. (4)	224. (2)	225. (4)	226. (1)	227. (2,3)	228. (2,4)
229. (3,4)	230. (1,4)	231. (3,4)	232. (4)	233. (1,2)	234. (1,3,4)
235. (3)	236.	237. (3)	238. (2)	239. (3)	240. (4)
241. (1)	242. (2)	243. (1)	244. (1)	245. (2,4)	246. (3,4)
247. (3)	248. (1,2)	249. (1,3)	250. (1,2)	251. (1,3)	252. (1,2,3,4)
253. (4)	254. (3)	255. (3)	256. (2)	257. (4)	258. (2)
259. (3)	260. (3)	261. (4)	262. (1,4)	263. (1)	264. (2,3)
265. (3)	266. (1,3,4)	267. (1,4)	268. (1,2,4)	269. (3)	270. (1,3,4)
271. (2)	272. (2)	273. (3)	274. (3)	275. (1)	276. (2)
277. (3)	278. (3,4)	279. (2)	280. (2)	281. (1,3)	282. (1,2,3,4)
283. (1,2,3,4)	284. (2,3,4)	285. (1,4)	286. (1,2)	287. (2,3)	288. (2)
289. (3)	290. (3)	291. (1)	292. (3)	293. (2)	294. (2)
295. (1,2)	296. (1,2)	297. (1,3)	298. (1,3,4)	299. (1,2,4)	300. (1,2,3,4)
301. (1,3,4)	302. (2,3,4)	303. (1,4)	304. (4)	305. (1,2,3)	306. (3)
307. (2)	308. (2)	309. (3)	310. (4)	311. (3)	312. (4)
313. (1,2)	314. (3)	315. (1,2,4)	316. (1,3)	317. (1,2,3,4)	318. (1,4)
319. (2,3)	320. (3,4)	321. (3)	322. (1,2,4)	323.(3)	324.(1)
325. (4)	326. (3)	327. (4)	328. (1,2,4)	329.(1,2,4)	330.(1,2,3,4)

331. (1)	332. (2,3)	333. (1,3)	334. (1,2,3)	335.(1,2,3)	336.(1,2,3,4)
337. (2,4)	338. (2)	339. (2)	340. (2)	341. (3)	342. (3)
343. (4)	344. (3,4)	345. (2,3)	346. (1,2,4)	347. (1,2)	348. (1,3)
349. (1,4)	350. (2)	351. (2)			